Algebraic Reasoning Student's of Islamic Junior High School Sabilurrosyad Malang in Solving Mathematical Problem's Based on Mathematical Ability

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Abstract: Algebraic reasoning forms the basis of all mathematical reasoning because in algebra, mathematical structures can be explored. This algebraic reasoning involves forming generalizations from previous experiences and skills related to numbers and calculations, formalizing these ideas with a symbol system and exploring the concept of a pattern and function. The aims of this study were (1) to determine the levels of algebraic reasoning students Islamic Junior High School Sabilurrosyad Malang in terms of high and low mathematical ability; and (2) to describe the characteristics of each level of algebraic reasoning students Islamic Junior High School Sabilurrosyad Malang in terms of high and low mathematical ability. This type of research is a case study qualitative research. The research subjects were selected from students Islamic Junior High School Sabilurrosyad Malang in the odd semester of the 2022/2023 academic year. The subject selection procedure used purposive sampling. The research data is in the form of algebraic reasoning characteristics, with data sources of the subject's occupation, interviews, and field notes. The research instrument was the researcher as the main instrument, written tests and interview guides as auxiliary instruments. Data collection techniques were carried out by task-based interviews. Testing the credibility of the data is done by giving assignments at different times (time triangulation). Data analysis uses a fixed comparison technique which generally consists of data reduction, data categorization, synthesis, and ends with developing a substantive theory. The results of this study are that there are four levels of algebraic students Islamic Junior High School Sabilurrosyad Malang, namely level 0, level 1 for students with low mathematical abilities and two students with high abilities who are at level above level 2 but have not yet reached level 3. Characteristics of algebraic reasoning in each level is at level 0: less able to understand the problem, uses natural language, which means students do not use variables or do not understand the meaning of variables, determine results depending on specific objects, cannot make generalizations so do not perform operations on variables in general forms. Characteristics of students with level 1 algebraic reasoning: can understand problems, can generalize using natural language, students cannot make general forms, so they do not perform operations on variables in general forms. While students with high mathematical abilities have the characteristics of students with level 2 algebraic reasoning but have not fully entered at level 3: able to understand problems, able to generalize and use symbolic language, general forms made are the result of generalizations using variables, able to make general forms is a function and performs operations on the variable after it is given a boost.

Keywords: Reasoning, Algebraic Reasoning, Levels of Algebraic Reasoning, Mathematical Ability

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Introduction

Algebra is a branch of mathematics. Algebra is a study of (1) the manipulation and transformation of symbolic statements, (2) the generalization of the rules about numbers and patterns, (3) the study of the structure and abstraction of systems from computations and relations, (4) the rules for transforming and solving equations, (5) learning about variables, functions, and expressing changes and relationships, (6) modeling mathematical structures. due to abstract mathematical objects, algebraic reasoning is needed (Watson, 2007: 8). Algebraic reasoning is a process that involves

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forming formulations or generalizations from experiences related to numbers and calculations, formalizing a mathematical idea using a system of symbols, and exploring concepts of patterns and functions. Thus, algebraic reasoning is important for students to have because it can help students understand mathematics beyond the results of specific calculations and procedural use of formulas (De Walle et al., Ontario Ministry of Education, 2013).

The main problem in students' algebraic reasoning is algebraic generalization (Radford, 2003). This algebraic generalization includes factual, contextual and symbolic generalizations. Generalization is an important aspect of algebraic reasoning which can then be used to develop profiles of students' algebraic reasoning levels. This research related to algebraic reasoning was carried out on junior high school students. This is because students of class VII junior high school start studying algebra, because at the elementary school level students have studied arithmetic (Kamol, 2010). According to Piaget's stages of cognitive development (Desmita, 2006), VII graders of junior high school (11 or 12 years old) are at the end of the concrete operational stage and at the beginning of the formal operational stage. The concrete operational stage is being able to draw logical conclusions based on the information provided to them even though the students' cognitive development is not perfect. The formal thinking stage is being able to think about abstract symbolic relationships, being able to imagine problems in his mind, and developing hypotheses logically.

In line with this research, it was found that during the transition from arithmetic to algebra, subject P (the best student in the study) showed various difficulties. Subject P can perform operations on zero, and regard it as a number. However, not knowing there were negative numbers, subject P thought the operation could not be performed when faced with subtracting a number with a smaller value from a number with a larger value. Research by Gallardo and Hernandez (2005) on the process of transition from arithmetic thinking to algebraic thinking in 16 students (age 12 to 13 years) in Mexico. This study investigates how students distinguish between the use of the equals sign (as an operator or expressing an equation), the minus sign (as an operator or as a negative sign), and the existence of zero.

Ake et al. (2013) proposed four primary levels of algebraic reasoning in their proceedings "Proto-Algebraic Levels of Mathematical Thinking". That is, level 0 algebraic students have not shown generalization and are still using arithmetic operations, level 1 algebraic students have tried to generalize but still use arithmetic language, level 2 algebraic students have been able to state equations with variables but have not been able to perform operations on variables, level 3 algebra is shown with the use of variables, can perform operations on these variables, and can be stated in the form of a function. The results of this research conducted on elementary school students showed that the highest level of the 52 selected student samples was level 2 algebra.

Based on the results of the pre-survey conducted to find out in general whether the problem exists or not. The pre-survey research was carried out by giving a written test regarding algebraic problems in the matter of number patterns. The test was given to FN as a student who was considered to represent the abilities of class VII students of Sabilurrosyad Islamic Middle School Malang. Analysis of answers and interviews with students shows that the characteristics of algebraic reasoning of FN students are that they can generalize, use symbolic language, that is, they can understand and use variables.

Based on the previous description and considering the diverse abilities of students, the researcher conducted research on the levels of algebraic reasoning in class VII students of SMP

Islam Sabilurrosyad Malang and the characteristics for each level of algebraic reasoning in class VII students of SMP Islam Sabilurrosyad Malang in solving mathematical problems. The diversity of student abilities is because each individual has a way and unit capacity in constructing mathematical knowledge. In this study used a review of mathematical abilities. The mathematical abilities are divided into high and low categories. The choice of upper and lower ability levels, because students with high and low ability levels have special characteristics, usually students with high abilities have special ways or tricks in solving mathematical problems. While students with low abilities, generally students with this ability need more time to understand the problems given. The aims of this study were 1) to determine the algebraic reasoning levels of students with high and low abilities in class VII Islamic Middle School Sabilurrosyad Malang in solving mathematical problems; 2) to find out the characteristics of each level of algebraic reasoning for class VII students of SMP Islam Sabilurrosyad Malang who have high and low mathematical ability in problem solving.

Materials and Methods

This research was conducted at Sabilurrosyad Islamic Middle School Malang. This madrasah is located within the Sabilurrosyad Gasek Malang Foundation. This school was chosen because the school has students with various intelligences, thus enabling researchers to obtain the data and information needed for research purposes. In addition, this school has never conducted research related to students' algebraic reasoning. This research is a qualitative research with a case study type. This is because, the purpose of this study is to determine the levels of algebraic reasoning and the characteristics of each level of algebraic reasoning of Sabilurrosyad Islamic Middle School students in Malang in solving mathematical problems in terms of mathematical abilities (high and low), based on the facts found as existence in the form of written, spoken, or observable actions. The subjects of this study were students of class VII Islamic Middle School Sabilurrosyad Gasek Malang in the odd semester of the 2022/2023 academic year. Subject selection was based on the following considerations, 1) class VII students have the ability to solve algebraic problems on number patterns; 2) students with various levels of mathematical ability, namely high and low, were selected with the aim of data diversity. As for the technique of selecting research subjects by purposive sampling.

The research data is in the form of students' algebraic reasoning characteristics based on high and low abilities which are obtained from the subject's work in solving problems about number patterns. Then the oral data interviews with research subjects after solving the problem. Sources of data in this study were obtained from the results of the subject's work, interviews with the subject, and field notes. The instruments in this study included the main research instruments, namely the researchers themselves as interviewers who were assisted by auxiliary instruments in the form of problem-solving test questions and interview guidelines. The instrument of problem solving test questions was validated before being used by professionals consisting of two lecturers and one teacher.

The data collection technique in this study was task-based interviews. In addition to collecting written data and interview results, students' behavior in solving problems was also observed. A tool is used in the form of a video recorder to facilitate data collection. After the data was collected, it was coded according to the algebraic reasoning level indicator proposed by Ake et al. (2013), then summarized the characteristics that appear. The subject's algebraic reasoning characteristics were then compared with the algebraic reasoning characteristics proposed by Ake et al. (2013), so that the subject's position in the level of algebraic reasoning can be known. The data obtained is used to answer what characteristics of algebraic reasoning can be observed from the symptoms that arise when the subject solves math problems. The credibility test in this study was carried out by time triangulation and increasing persistence. The data analysis technique was carried out using the Constant Comparative Method, which is to constantly compare categories with other categories (Glaser and Strauss in Moleong, 2013). In general, the data analysis process includes: data reduction, data categorization, synthesis, and ends with developing a working hypothesis which is a substantive theory.

Results and Discussion

Subject selection was carried out by purposive sampling, namely taking each of the 2 research subjects according to the teacher's directions from each ability level, namely 2 students with high mathematical abilities and 2 students with low mathematical abilities. Data collection techniques were carried out using task-based interviews, namely giving assignments in the form of mathematical problems and then confirming answers through interviews. Research data must be reliable or credible, for this reason researchers triangulate time. The researcher conducted task-based interviews twice at different times, but if the data obtained was invalid, another task-based interview was conducted at different times. The validity of the data on time triangulation is that the data obtained at different times does not show a significant difference.

The research data collected is the characteristics of algebraic reasoning obtained from interviews based on number pattern problem solving tasks. The assignment in the form of a written test was first given to predetermined subjects, then analyzed so that the algebraic reasoning characteristics of each subject were known. On a different day the researcher gave the subject a second written test assignment, the data was analyzed and checked to find out whether there was a difference with the first data. When there is a difference in algebraic reasoning characteristic data between the first and second tasks, the researcher gives the third task. Only subjects whose data showed inconsistencies were given the third task. Then the data for each subject obtained based on the first, second, and even third task-based interviews were compared. Subject data is said to be valid if there are no more differences between the data in the first, second, and third tasks.

Subject data that has been declared valid is then analyzed further and then compared with the level of algebraic reasoning Ake et al. (2013). Ake et al. (2013) proposed four levels of algebraic reasoning using the following three criteria 1) there is a general form resulting from the generalization process; 2) Steps in generalizing; 3) Operations and transformations on variables in the general form resulting from the generalization process. Based on these criteria, Ake et al. proposed four levels of algebraic reasoning. Characteristics of each algebraic level according to Ake et al. (2013) is described in the following table.

Table 1 Algebraic Reasoning Levels Ake *et al.*

Level	Characteristics
Level 0	- extensive object - expressed in language as it is, numeric, iconic, and with certain gestures - there is a symbol (still an image) to represent a value - the results obtained are from operations on special objects
Level 1	- intensive object (intensive object) - generalizations can be clearly recognized by language as it is, numeric, iconic, and with specific gestures - there are symbols that refer to intensive objects, but do not perform operations on these objects
Level 2	- involve variables declared in a symbolic language that refer to intensive objects, but are still temporary - the general form is the equation Ax±B=C - do not perform operations with variables to create a general shape
Level 3	- Intensive objects are expressed by the language of symbols - perform transformations without changing the equation (equivalent) - there are operations on variables to create general forms

(source: Ake *et al.*, 2013)

By using the task-based interview method, the following results are obtained. Students with the initials RA are students with low mathematical abilities. The answers from these students are as follows.

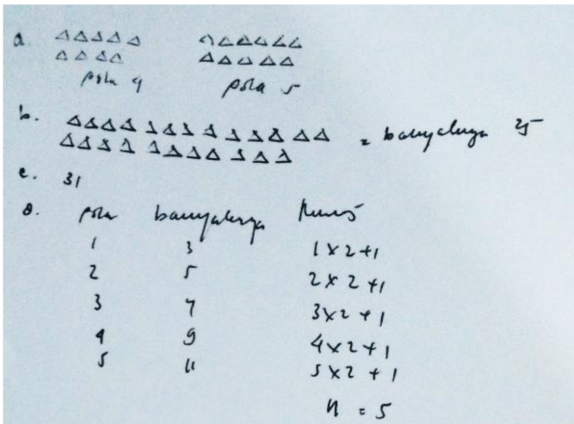

Figure 1. Answers subject with low math ability

Based on the student's answers, then analyzed using indicators of algebraic reasoning Ake et al.

(2013), namely 1) understanding the problem. At this stage the subject reads the problem given, the subject observes changes in the arrangement of tiles, then counts the number of tiles and begins to think about the next pattern arrangement. The subject determines the number of tiles in the pattern sequence asked by using pictures, sorting the patterns one by one from the known patterns. Thus, it can be understood that the subject uses the information contained in the problem to determine how the problem can be resolved. So it is concluded that the subject understands the given mathematical problem; 2) Create a settlement plan. At this stage, the subject raises variable n, but the subject does not understand the meaning of variable n, meaning that the subject still uses natural language. When the subject does not understand the meaning of the written variable, it means that the subject does not use symbols. Furthermore, the subject observed the shape and number of arrangements in each pattern, then to determine the number of tiles in a pattern in a certain order, the subject still manually counted the number of tiles from the previous pattern, meaning that the subject still depended on a particular pattern. The subject counts the number of tiles using the picture first and then counts the number of tiles in a certain order. The subject can determine the calculation but the subject does not understand n as a variable. This means that the subject does not generalize; 3) complete the problems. In this stage, the subject makes a general form but does not understand its meaning and cannot make generalizations. In addition, the subject also does not perform variable operations using this general form, meaning that the subject cannot create a general form and does not perform variable operations on the general form; 4) solve the problem. At this stage, the subject uses the image to determine the number of tiles but when determining the number of tiles in a pattern with high order it cannot be done with images. That is, the subject cannot determine the number of tiles in the 200th pattern. This shows that the subject did not solve the problem and because the subject also did not understand the formula he made, the subject did not solve the problem with the general form.

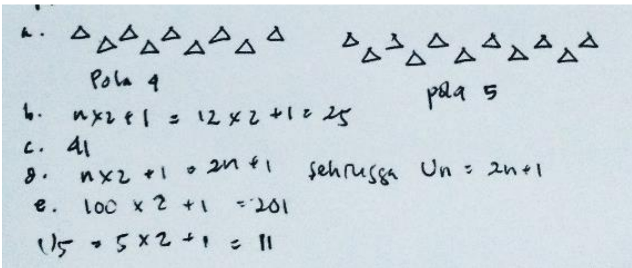

Figure 2. Answers subject with high math ability

While the research results for NS subjects (age 12 years), the following results were obtained 1) NS subjects read the problem given, then counted the number of tiles in each known pattern. By knowing the number of tiles that make up the pattern, the subject then conducts a trial and error to determine the right calculation to find out the number of tiles in each pattern. Thus, the subject uses the information in the problem to determine the steps to be taken to solve the problem. So that it can be seen that the subject understands the problem; 2) Make generalizations. At this stage the subject uses the letter n as a variable. This shows that the subject no longer uses natural/as-is language; the subject states the formula for determining the number of tiles in question. This indicates that the subject does not depend on a specific object/pattern to determine the number of tiles in question; The subject knows the meaning of the variable shown when the subject replaces n with 10 when determining the number of tiles in the 10th pattern. This shows that the subject knows that n represents a sequence of patterns. Based on this explanation, it shows that the subject has used symbolic language; subject uses a formula to determine the number of tiles in the tenth pattern. The formula is obtained by the subject by paying attention to the regularity of the pattern. This means that the subject has generalized; 3) Create a general shape. At this stage, the subject generalizes, by writing $Un = 2n + 1$ as the general formula/form stated by the subject to answer further questions. The subject understands the meaning of the variables in the general form that is made, meaning that the subject makes the general form; the subject can perform operations on variables in the general form after getting encouragement when the researcher asks. This means that the subject has the ability to operate on variables, but is not yet skilled at using that ability; 4) Solve the problem. At this stage, the subject can determine the number of tiles in the hundredth pattern. So that it can be seen that the subject can solve the problem, the subject determines the number of tiles in the hundredth pattern with the formula stated by the subject to answer the next question. The subject replaces the n variable with 100, so the subject knows how many tiles are needed to make the hundredth pattern. This shows that the subject understands the general form that is made, which is shown by using the general form to solve the problem. Thus, NS, who is a student with high mathematical ability, has algebraic reasoning level 2 but has not yet reached level 3.

Discussion

Based on the results of this study, it can be seen that students with early low math skills have level 0 algebraic reasoning. With the following characteristics, 1) with level 0 algebraic reasoning determines results depending on certain patterns (special objects), using natural language/as it is , does not understand the use of variables, cannot create general forms, and cannot perform operations on variables. This shows that students are still in the concrete operational stage and use factual generalizations. The characteristics of students with low ability algebraic reasoning are as follows 1) Understanding problems with characteristics, namely; a) using the information in the problem to determine the steps to solving the problem, this shows that students understand the problem. At the generalization stage, the subject uses natural language/as it is, determines the quantity of patterns by drawing and hanging with special objects, does not use symbolic language (not categorized), does not generalize (is not characterized); 3) Create a general form with variables. At this stage, the subject cannot make general forms using variables (uncategorized, and does not perform variable operations. There are differences in the algebraic reasoning characteristics of class VII students of SMP Islam Sabilulrosyad Malang with the theory of Ake et al. (2013) at level 3, so that the algebraic reasoning level of class VII students of SMP Islam Sabilulrosyad Malang cannot be said to be at level 3. The difference with theory is that students have the ability to perform operations on variables, but need encouragement to do so. Based on an analysis of the characteristics of their reasoning, class VII students of Sabilulrosyad Islamic Middle School Malang (11 to 13 years old) with algebraic reasoning levels that are above level 2 but have not yet reached level 3 can control variables, test hypotheses, and are able to draw conclusions in the form of general made. This is in accordance with the theory of cognitive development that students aged 11 to.d. 12 years to adulthood are in the formal operational stage. Based on the type of generalization, students use symbolic generalization.

The levels proposed by Ake, et al (2013) use the following three criteria: 1) there is a general form resulting from the generalization process; 2) Steps in generalizing; 3) Operations and transformations on variables in the general form resulting from the generalization process. Generalization is part of algebraic reasoning that develops through continuous experience. This was revealed in Radford's (2003) research on solving number pattern problems. Students are not necessarily able to generalize about number pattern problems, generalizations develop from calculations with concrete numbers to the use of symbols. Radford (2003) identifies the development of generalization in three types, namely factual generalization, contextual generalization, and symbolic generalization. The factual generalization type is a generalization of mathematical objects that use a numerical scheme that is limited to the level of concrete numbers. Contextual generalization is the next type of generalization that has left calculations with concrete numbers, the determination of values is done by paying attention to the before and after patterns without involving specific patterns. Generalizations that have used letters as symbols in determining values, and determining the simplest form of the formula are called symbolic generalizations.

Conclusions

Based on an analysis of the characteristics of algebraic reasoning for class VII students of SMP

Islam Sabilurrosyad Malang, there are two levels of algebraic reasoning, namely level 0 and the level between level 2 and level 3. The data was obtained from an analysis of four subjects, namely two subjects with low mathematical ability in algebraic reasoning level 0, two subjects with high mathematical abilities at level algebraic reasoning between level 2 and level 3. The characteristics of level 0 algebraic reasoning are that students can understand the problem given. Students use natural language / as it is, students get quantity in certain patterns depending on the previous pattern, namely continuing the image from the previous pattern. Students cannot create common shapes using variables. Students do not perform variable operations on general forms, students cannot solve problems. Students cannot make general forms, so the subject does not use general forms to solve problems. While the characteristics of level algebraic reasoning between level 2 and level 3 (not yet reached level 3) are as follows. Students can understand the problems given. Students use variables and know their meaning, so that it can be said that students use symbolic language. Students pay attention to the arrangement of images and the quantity of each sequence in a known pattern, then students can determine the calculation to determine the quantity in the pattern in question. This shows students can make generalizations. Students can create common forms using variables and know their meaning. Students can perform variable operations on general forms after getting encouragement (not yet fully categorized. Students can solve problems. Students understand the general forms that are made, which is shown by using these general forms to solve problems.

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