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TEACHER STUDENT'S METACOGNITIVE FAILURE WHEN SOLVING SPHERE EQUATION QUESTION AND THEIR SCAFFOLDING

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Abstract

Problem solving is the core of mathematics. To determine one's problem-solving performance, metacognition is frequently used. Instead of success, metacognition failure could occur and scaffolding is needed to achieve correct solution. Three selected participants are students of Mathematics Education Department of UIN Maulana Malik Ibrahim Malang, due to their low-level question-solving performance in Task 1. This qualitative research aims to describe these three participants' metacognitive failure and their scaffolding. Task 1 answer-sheet and Task 1-based interview from each of the participants are triangulated as main data. The result of this research indicates metacognitive blindness, metacognitive mirage, and metacognitive vandalism occur variously from these participants. Based on metacognitive failure which occur, the proper scaffoldings are chosen. They are reviewing, restructuring, and making connection from the question given to help participants achieve correct solution. Task 2, equivalent with Task 1, is given to all three participants to observe whether these participants are making no metacognition failure or no need for more scaffolding. This research provides practical example in using combination from task answer sheet and task-based interview to assess students' metacognition, particularly metacognitive failure. In conclusion, after students' metacognitive failure is identified, proper scaffolding support may be applied and variously. Then, better mathematics learning quality can be achieved.

Keywords: metacognition; metacognitive failure; scaffolding.

Abstrak

Pemecahan masalah adalah inti dari matematika. Untuk menentukan performa pemecahan masalah seseorang, metakognisi sering digunakan. Alih-alih sukses, kegagalan metakognisi dapat terjadi dan perancah diperlukan untuk mencapai solusi yang benar. Tiga partisipan terpilih merupakan mahasiswa Jurusan Pendidikan Matematika UIN Maulana Malik Ibrahim Malang karena performa penyelesaian soal di Tugas 1 yang rendah. Penelitian kualitatif ini bertujuan untuk mendeskripsikan kegagalan metakognitif ketiga partisipan dan perancahnya. Lembar jawaban Tugas 1 dan wawancara berbasis Tugas 1 dari setiap peserta di triangulasi sebagai data utama. Hasil penelitian ini menunjukkan kebutaan metakognitif, fatamorgana metakognitif, dan perusakan metakognitif terjadi secara beragam dari para peserta ini. Berdasarkan kegagalan metakognitif yang terjadi, perancah yang tepat dipilih. Perancahnya yaitu meninjau, merestrukturisasi, dan membuat koneksi dari pertanyaan yang diberikan untuk membantu peserta mencapai solusi yang benar. Tugas 2, setara dengan Tugas 1, diberikan kepada ketiga peserta untuk mengamati apakah peserta ini tidak membuat kegagalan metakognisi atau tidak perlu perancah lagi. Penelitian ini memberikan contoh praktis dalam menggunakan kombinasi dari lembar jawaban tugas dan wawancara berbasis tugas untuk menilai metakognisi siswa, khususnya kegagalan metakognitif. Sebagai simpulannya, setelah kegagalan metakognitif siswa diidentifikasi, dukungan perancah yang tepat dapat diterapkan dan dapat beragam. Lalu, mutu pembelajaran matematika yang baik dapat dicapai.

***Kata kunci**: Kegagalan metakognitif; metakognisi; perancah.*

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INTRODUCTION

Problem solving performance is one of the main competencies in 21st century skill (Lin, Yu, Hsiao, Chang, & Chien, 2020; Yusoff, Ashaari, Wook, & Ali, 2020). To answer the problem or question will lead to the metacognitive process. Wilson & Clarke (cited in Kuzle, 2018) define metacognition as one's awareness, regulation, and evaluation of his/her thinking. The main feature of the metacognitive process is that problem solver can aware with appropriate strategies, mathematical notations, logical reasons (Güner & Erbay, 2021) and problem-solving performance (Zhao et al., 2019). In fact, assessing students' metacognition become rare activity to do. Further, giving scaffold into students' metacognition failure is needed to improve students' problem-solving performance (Matsuda, Weng, & Wall, 2020).

Scaffolding is defined as the process that assisting learner to solve problem, performing a task, or achieving a goal which would be beyond his unassisted efforts then remove it when learner can do it by himself (Bakker, Smit, & Wegerif, 2015). Different contexts of scaffolding are peer scaffolding (Haataja et al., 2019), whole-class setting (Abdu, Schwarz, & Mavrikis, 2015), and technological support (Albano & Dello Iacono, 2019). Proper scaffolding can be given after identifying metacognitive failure which occurs.

There are two kinds of method to assess one's metacognition when solving problem, *online* assessment or *offline* assessment (Muncer et al., 2022; Veenman & van Cleef, 2019). *Online* assessment is a method when a person conveys immediately what he/she think during problem solving but it is called *offline* assessment when a person conveys after problem solving. Further, both online and offline assessment has advantages and disadvantages (Kuzle, 2018). To deepen the analysis of one's metacognition, there are several problem-solving frameworks can be chosen. They are Polya's Stages, Schoenfeld's episode, Garofalo & Lester's stages, Artzt & Armour-Thomas' episode (Sozen Ozdogan, Ozçakir, & Orhan, 2019) and five phase cognitive-metacognitive framework is introduced by Yimer & Ellerton (Muhali, Yuanita, & Ibrahim, 2019). Further, metacognitive failure can be analyze by red-flag framework (Huda & Marsal, 2021) or assimilation-accommodation framework (Nizlel et al., 2016)

Quite different with previous research, this research takes three different aspects. Firstly, this research study one's metacognitive based on Yimer & Ellerton (cited in Muhali et al., 2019) cognitive-metacognitive framework and combine it with task-based interview situation. Secondly, multivariable calculus is known by student as difficult course. Thirdly, red flag metacognitive failure analysis is chosen in this research. These three aspects are chosen since mostly educators (lecturer and teacher) has no practical guide to improve their students' metacognition.

This research provides best practice for educators to assess their students' metacognition effectively and efficiently. By paying attention to how students think, educators can provide proper scaffolding so that later students can improve their performance with their own efforts (Cevikbas, Kaiser, & Schukajlow, 2022). As a result, better quality in mathematics learning is achieved.

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RESEARCH METHODS

This research aims to explore metacognition failure which consider as the cause of poor mathematics performance and follow it up with scaffolding. Therefore, this research is categorized as qualitative research (Creswell, 2014). The research procedure consisted mainly in two steps. First step, identifying participant's metacognition failure. To do so, to collect participant's metacognition failure, each participant was asked to answer the Task I given. Second step, constructing scaffolding by specific metacognition failure which occur for each participant. Later, each participant was asked to do Task 2 to check scaffolding effectiveness.

This research is conducted in Mathematics Education Department of Maulana Malik Ibrahim Malang State Islamic University. The participants candidate in this research were taken from thirty teacher-students who enroll Multivariable Calculus in 2022/2023 academic year. The participants in this research were selected by purposive sampling technique which follow these procedures:

1. Giving Task I that must be answer by thirty participants candidate.
2. Assessing participants candidate's problem-solving performance.
3. Considering participant candidate who perform low-level problem-solving performance.

By those procedures, three participants from the lowest problem-solving performance in Task 1 were selected.

This research has two kinds of instruments, they are main and supporting instrument. The main instrument in this research is the researcher itself because researcher acts as a planner, data collector, data interpreter, and research reporter. The supporting instruments were Task 1, Task 2, and task-based interview protocol which validated by researcher's colleague by three aspects, they are content, construct, and language aspect. The Task 1 and 2 can be seen in Figure 1 and 2.

TASK 1

1. Given points $P_1(0,0,0)$ and $P_2(2,-2,-2)$. Determine the distance of these points!
2. Given points $P_1(-2,0,2)$ and $P_2(0,1,0)$. Determine general sphere equation with P_1 as its center and trough P_2!

Figure 1. Task-1 instrument

TASK 2

1. Given points $P_1(1,0,-1)$ and $P_2(1,3,3)$. Determine general sphere equation with P_1 as its center and trough P_2!

Figure 2. Task-2 instrument

Task 1 and Task 2 (Figure 1 and Figure 2) were two equivalent tasks with the same topic, determining general sphere equation from two points given. Each task has two main steps, they are: 1) finding the sphere radius by using distance formula from two points given, and 2) substituting the radius in the first step into general sphere equation. Participants were asked to answer Task 2 to check whether scaffolding was successful.

To deepen the analysis of participants' metacognition, five-phase cognitive metacognitive framework by Yimer & Ellerton (cited in Muhali et al., 2019) was applied (see Table 1). By using this framework together with task-based interview, participants' metacognition failure can be identified. After gaining valid data with respect to participants' metacognitive failure, scaffolding is constructed by each participants' metacognitive failure.

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Table 1. Five-phase cognitive-metacognitive model Yimer & Ellerton (cited in Muhali et al., 2019)

Problem-Solving Phase	Subphase	Code
Engagement (1)	A. Initial understanding (noting main ideas and/or making pictures)	1A
	B. Information analysis (information recognition, key information identification which relevant to solve the problem, relating it to a specific mathematical domain)	1B
	C. Reflecting on the problem (assessing familiarity or remembering similar previously solved problem, assessing difficulty level, assessing the necessary store of knowledge one has in relation to the problem)	1C
Transformation-Formulation (2)	A. Exploring (using particular cases or numbers to visualize the situation in the problem)	2A
	B. Conjecturing or hypothesizing (based on specific observations and prior experiences)	2B
	C. Reflecting feasibility of the exploration and conjecturing	2C
	D. Formulating a plan (arranging a strategy either to test conjectures or devising global or local plans)	2D
	E. Reflecting on the feasibility of the plan based on the key features of the problem	2E
Implementation (3)	A. Exploring key features of plan	3A
	B. Assessing the plan with the conditions and requirements set by the problem	3B
	C. Implementing the plan (doing action by either computing or analysing)	3C
	D. Reflecting on the appropriateness of actions	3D
Evaluation (4)	A. Rereading the problem whether the result has answered the question in the problem or not	4A
	B. Assessing the plan for consistency with the key features as well as for possible errors in computation or analysis	4B
	C. Assessing for reasonableness of results	4C
	D. Making a decision to accept or reject a solution	4D
Internalization (5)	A. Reflecting on the entire solution process	5A
	B. Identifying critical features in the process	5B
	C. Evaluating the solution process for adaptability in other situations, different ways of solving it, and elegance	5C
	D. Reflecting on the mathematical rigor involved, one's confidence in handling the process, and degree of satisfaction	5D

RESULT AND DISCUSSION

Participants' Cognitive-Metacognitive Aspect

Based on the interview data, it can be observed that all participants who perform low-level problem-solving has a tendency to act 1C dan 2E (see table 2). 1C means that participants tried to reflect the question given. Further, 2E means that participants tried to reflect

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on the feasibility of the plan based on the key features of the problem. As a result, they were forced to rethinking their engagement with the question and transforming or formulating the question several time. This finding is similar to Adinda et al., (2021) who explain one's experiencing obstacle in solving problem due to lack of attention to the problem and fail to recall required knowledge.

Table 2. Participants' cognitive-metacognitive action

Subject	Cognitive-Metacognitive Action
S1	1C, 2E, 3D, 4D
S2	1C, 2E, 3B, 3D, 4C
S3	1B, 1C, 2E, 2D, 4C

S1 tried to assess familiarity of the question given **[1C]** and reflected on the feasibility of the plan based on the key features of the question given **[2E]**. After that, S1 reflecting his appropriateness of his action **[3D]** and reject his solution **[4D]**.

Similar to S1, S2 tried to assess the necessary store of knowledge that relate to the problem **[1C]** and reflected on the feasibility of the plan based on the key features of the question given **[2E]**. However, S2 felt doubt on his mathematical model **[3B]** and unsure with his action. **[3D]**. Further, S2 did not assess his reasonableness of his result **[4C]**.

S3 has managed to identify the goal of the question given **[1B]**. Next, S3 tried to assess familiarity of the question given and assess the necessary store of knowledge that relate to the problem **[1C]**. S3 also tried to reflect on the feasibility of the plan based on the key features of the question given **[2E]** and promoted 2 local plans **[2D]**. S3 assessed his reasonableness of his result **[4C]**, therefore he confused.

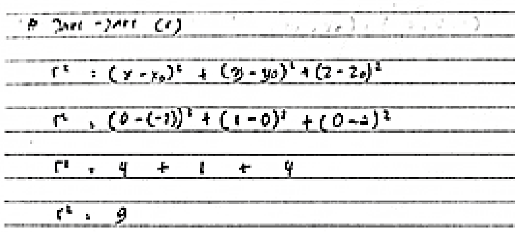
Participants' Metacognitive Failure

Based on the participants' answer sheet, it can be observed that in this research occur two kinds of metacognitive failure (see table 3). S1 and S2 were experiencing mirage metacognitive failure. In contrast, S3 were experiencing blindness metacognitive failure. Goos (cited in Huda & Marsal, 2021) describes that the participants' metacognitive failure can be caused by error recognition, lack of progress in the process of finding solution, or anomalous results.

Table 3. Participants' Metacognitive Failure

Subject	Metacognitive Failure
S1	Mirage
S2	Mirage
S3	Blindness

Answer Sheet



Translation

Radius (r)

$r^2 = (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2$

$r^2 = (0 - (-2))^2 + (1 - 0)^2 + (0 - 2)^2$

$r^2 = 4 + 1 + 4$

$r^2 = 9$

Figure 3. S1 Task-1 answer-sheet

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Based on Task-1 Answer Sheet (Figure 3) from S1 above, it can be observed that S1 has assessed the question difficulty **[1C]**, tried to formulate a plan but has no global planning **[2D]**, and tried to determine feasibility of the question **[2E].** S1 successfully initializing the formula to find radius. S1 feels that his performance doesn't suit his plan **[3D]**. However, after achieve r^2=*9*, S1 cannot continue any further and feel he didn't solve the question very well **[4D]**. Hence, r^2=*9* become red flag because S1 committing no wrong calculation but doubtful to apply the result into next plan. This finding is described as *metacognitive mirage* situation (Huda & Marsal, 2021; Rozak, Subanji, Nusantara, & Sulandra, 2018).

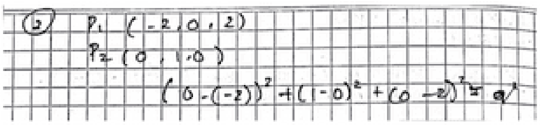
Answer Sheet	Translation
	2. P_1 $(-2, 0, 2)$ P_2 $(0, 1, 2)$ $(0-(-2))^2 + (1-0)^2 + (0-2)^2 = a^2$

Figure 4. S2 Task-1 answer-sheet

Based on Task-1 Answer-Sheet (Figure 4) from S2, it can be observed that S2 has some difficulties. S2 tried to assess the feasibility of his plan **[2E]** and try to recall related knowledge. S2 assessing the plan with the conditions and requirements set by the problem **[3B]**. S2 is not confident the model/representation he made **[3D].** S2 did not assess for reasonableness of his result. Further, general sphere equation become red flag since S2 can correctly recall general sphere equation but cannot identify what are the unknowns to carry on into planning phase. This finding is described as *metacognitive mirage* situation (Huda & Marsal, 2021; Rozak et al., 2018).

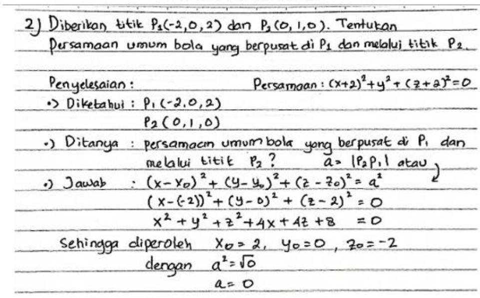
Answer Sheet	Translation
	2. Given point P_1 $(-2, 0, 2)$ and P_2 $(0, 1, 0)$. Determine General sphere equation which centre P_1 in and through P_2 Solution: Equation: $(x+2)^2 + y^2 + (x+2)^2 = 0$ Known: P_1 $(-2, 0, 2)$ P_2 $(0, 1, 0)$ Asked: general sphere equation which centre in P_1 and through P_2? $a = \|P_1P_2\|$ or Answer: $(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = a^2$ $(x-(-2))^2 + (y-0)^2 + (z-2)^2 = 0$ $x^2 + y^2 + z^2 + 4x + 4z + 8 = 0$ So, we get: $x_0 = 2,\ y_0 = 0,\ z_0 = -2$ with: $a^2 = \sqrt{0}$ $a = 0$

Figure 5. S3 Task-1 Answer-Sheet

Based on Task-1 Answer Sheet (Figure 5) from S3, it can be observed that S3 has done many cognitive-metacognitive activities. In orientation phase, S3 has successfully done in analyzing the information and conditions given **[1B]**, assessing familiarity with the question given **[1C]**

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and mentioning correct general sphere equation **[1C]**. In organization phase, S3 has assess the feasibility of his plan by arranging global planning **[2E]** and local planning **[2D]**. In verification phase, S3 feels confuse with the local result he got whether it is suit with his plan or problem condition **[4C]**. S3 can relate $|P_1P_2|$=a which is radius of the sphere is determined by distance from $|P_1P_2|$. However, S3 does a mistake by thinking the result a=0. This mistake is carried out into wrong conclusion of sphere equation $(x+2)^2 + y^2 + (z+2)^2 = 0$. This finding is described as *metacognitive blindness* situation (Huda & Marsal, 2021; Rozak et al., 2018) Metacognitive blindness can be caused by lack of understanding relate to the question given, forgetting a procedure, writing information incorrectly, being careless, and guessing.

Participants' Scaffolding based on their Metacognitive Failure

Based on the participants' metacognitive failure, it can be observed that in this research participants who experiencing mirage metacognitive failure have a tendency where reviewing scaffolding occurs more often (S1 and S2). On the other side, participant who experiencing blindness metacognitive failure have a tendency where restructuring scaffold-ding occurs more often (S3). Table 4 describes participants' scaffolding.

Table 4. Participants' Scaffolding

Subject	Metacognitive Failure	Scaffolding Sequence
S1	Mirage	Reviewing, Restructuring, Reviewing, and Making Connection
S2	Mirage	Reviewing, Making Connection, Restructuring, and Making Connection
S3	Blindness	Reviewing, Restructuring, Restructuring, and Making Connection.

Based on Task-1 and interview, for S1, scaffolding type reviewing, restructuring, and making connection were chosen (Wulan, Subanji, & Muksar, 2021). In the *reviewing* interaction, it shows us that S1 is engaged with the question. S1 succeed in identifying the goal of question number 2 was to find spere equation that it's center in P_1. In the *restructuring* interaction, it shows us that S1 can identify the goal and what unknown is missing to complete the sphere equation. S1 successfully managed to calculate the sphere radius. *Making connection* interaction is used when S1 need to apply the result (radius) into general sphere equation. S1 successfully found that the general sphere equation was $(x+2)^2-(y-0)^2-(z-2)^2=3^2$.

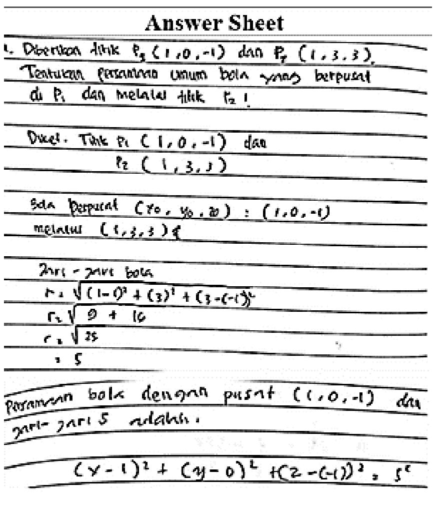

Figure 4 (a). S1 task-2 answer-sheet

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Translation

Given points P_1 $(1, 0, -1)$ and P_2 $(1, 3, 3)$
Determine general sphere equation which center in P_1 and through point P_2!

Known point P_1 $(1, 0, -1)$ and
P_2 $(1, 3, 3)$

Sphere centre $(x_0, y_0, z_0) = (1, 0, -1)$
Through $(1, 3, 3)$

Sphere radius
$r = \sqrt{(1-1)^2 + (3)^2 + (3-(-1))^2}$
$r = \sqrt{9 + 16}$
$= 5$

Sphere equation with centre $(1, 0, -1)$ and Radius 5 is

$(x-1)^2 + (y-0)^2 + (z-(-1))^2 = 5^2$

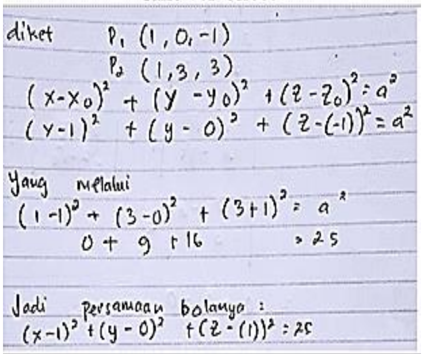
Figure 4 (b). The translation of S1 task-2 answer-sheet

To check the effectiveness of the scaffolding, S1 is asked to do Task-2 (Figure 4). From S1's Task-2 Answer Sheet, it is concluded that no metacognition failure occurs.

Based on Task-1 and interview, for S2, scaffolding type reviewing, restructuring, and making connection were chosen (Wulan, Subanji, & Muksar, 2021). In the *reviewing* interaction, it shows us that S2 knows the goal of the question given. S2 successfully mention the goal of the question was seeking sphere equation. In the restructuring interaction, S2 can identify what is the unknown which is missing to complete the sphere equation. S2 could determine the radius of the spere. *Making connection* interaction is used when S2 need to apply the result (radius). S2 successfully implemented the radius into sphere equation. S2 answered the sphere equation correctly *(x+2)²-(y-0)²-(z-2)²=3²*.

To check the effectiveness of the scaffolding, S2 is asked to do Task-2 (Figure 6). From S2 Task-2 Answer Sheet, even though S2's didn't provide good explanation, it is concluded that S2 has right path to the solution. Furthermore, no scaffolding is needed.

Answer Sheet



Translation

Known P_1 $(1, 0, -1)$
P_2 $(1, 3, 3)$
$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = a^2$
$(x-1)^2 + (y-0)^2 + (z-(-1))^2 = a^2$

Which through
$(1-1)^2 + (3-0)^2 + (3+1)^2 = a^2$
$0 + 9 + 16 = 25$

So, sphere equation:
$(x-1)^2 + (y-0)^2 + (z-(-1))^2 = 25$

Figure 6. S2 Task-2 Answer-Sheet

Based on Task-1 and interview, for S3, scaffolding type reviewing, restructuring, and making connection were chosen (Wulan, Subanji, & Muksar, 2021). In the *reviewing* interaction, it shows us that S3 knows the goals of the question given. S2 could determine the aim of the question was to find general solution of sphere with certain center point (P_1) and through another point (P_2). In the restructuring interaction, it shows us that S3 can identify what unknown is missing to complete the sphere equation (radius). *Making connection* interaction is used when S3 need to remind the distance of P_2P_1 is also become the sphere radius.

To check the effectiveness of the scaffolding, S3 is asked to do Task-2 (Figure 8). From S3's Task-2 Answer Sheet, we conclude that no metacognition failure occurs.

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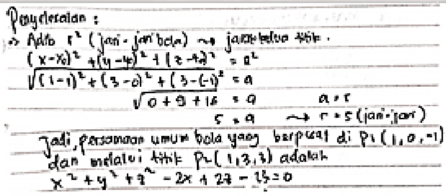
Answer Sheet	Translation
	Solution: Want to proof r^2 (sphere radius) → distance from two points $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2 \quad = a^2$ $\sqrt{(1-1)^2+(3-0)^2+(3-(-1))^2} = a$ $\sqrt{0+9+16} = a \quad a = r$ $5 = a \rightarrow r = 5$ (radius) So, general sphere equation whose centre in $P_1(1,0,-1)$ and through point $P_2\ (1,3,3)$ is $x^2+y^2+z^2-2x+2z-23=0$

Figure 8. S3 Task-2 Answer-Sheet

CONCLUSION AND SUGGESTION

This research has described three teacher students' metacognitive failures analysis that contribute to their low-level problem-solving performance. In mathematics teaching and learning context, this research implies that each student who perform low-level problem-solving may experience metacognitive failure which may occurred in different condition. To improve the quality of mathematics teaching and learning, metacognitive assessment for student become essential.

After educator (teacher or lecturer) knowing students' metacog-nition, particularly metacognitive failure, appropriate scaffolding can be generated. However, effective scaffolding scenario could depend on students' performance or work. To improve problem solving performance, using red flag as scaffolding starting-point is considered effective. Educator can give reviewing-scaffolding, restructuring-scaffolding, and connection-making scaffolding in different sequence (various scenario). When students start to recognize red flag from their own work and how to deal with it, educator can decrease or even eliminate the scaffolding.

There are two domains to explore this research further. First, next researcher can try to use different protocol in assessing students' metacognition (i.e., self-report protocol instead of interview). Second, next researcher can try to design effective scaffolding (i.e., peer scaffolding or technological-based scaffolding instead of tutor-tutee scaffolding).

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