InPrime: Indonesian Journal of Pure and Applied Mathematics
Vol. 6, No. 2 (2024), pp. 112 – 123, doi: 10.15408/inprime.v6i2.41438
p-ISSN 2686-5335, e-ISSN: 2716-2478

Optimal Control of a Modified Mathematical Model of Social Media Addiction

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Abstract

This research investigates the application of optimal control in the Susceptible, Exposed, Addicted, Recovery, Quit (SEA_1A_2RQ) model to address social media addiction. The primary objective is to develop an effective control strategy to reduce the prevalence of social media addiction. The methodology employs Pontryagin's maximum principle to formulate the optimal control problem, incorporating two time-dependent control variables: control (u_1) and treatment (u_2). The optimal control model is numerically simulated using the 4th-order Runge-Kutta method. Comparative analysis of the simulation results, with and without control, demonstrates significant differences in all population groups after four years. The findings reveal that implementing control (u_1) and treatment (u_2) markedly decreases the number of individuals addicted to social media, highlighting the efficacy of the proposed strategy in mitigating social media addiction.

Keywords: optimal control; social media addiction model; Pontryagin maximum principle.

Abstrak

Penelitian ini mengkaji penerapan kontrol optimal pada model Susceptible, Exposed, Addicted, Recovery*, model* Quit (SEA_1A_2RQ) *untuk menangani kecanduan sosial media. Tujuan utama penelitian ini adalah untuk mengembangkan strategi kontrol yang efektif untuk mengurangi jumlah individu yang mengalami kecanduan media sosial. Metodologi penelitian ini menggunakan prinsip maksimum* Pontryagin *untuk merumuskan masalah kontrol optimal, yang menggabungkan dua variabel kontrol bergantung pada waktu: pengendalian* (u_1) *dan pengobatan* (u_2)*. Model kontrol optimal disimulasikan secara numerik menggunakan metode Runge-Kutta orde 4. Analisis komparatif dari hasil simulasi, dengan dan tanpa kontrol, menunjukkan perbedaan yang signifikan pada semua kelompok populasi setelah empat tahun. Temuan tersebut mengungkapkan bahwa penerapan pengendalian* (u_1) *dan pengobatan* (u_2) *secara signifikan mengurangi populasi pengguna kecanduan sosial media, yang menyoroti kemanjuran strategi yang diusulkan untuk mengurangi kecanduan sosial media.*

Kata Kunci: *kontrol optimal; model kecanduan media sosial; prinsip maksimum* Pontryagin.

2020MSC: 49N90, 91D10, 92B05.

1. INTRODUCTION

Information and communication technology (ICT) advances have increased recently and affected how people interact. In this era, most people use mobile phones or gadgets to access the internet. Technological advancements provide easy and quick access to various social media platforms for sharing and communicating through the internet. Users can easily interact with many people and expand relationships without considering distance and time because information is disseminated quickly and efficiently [1]. When viewed from the purpose of using the internet, the Central Statistics

* Corresponding author
Submitted September 23rd, 2024, Revised October 22nd, 2024,
Accepted for publication October 24th, 2024, Published Online October 28th, 2024

Agency (BPS) in 2023 stated that 84.37% of teenagers aged 16-30 used the internet to access social media [2]. The social media platform currently attracting attention is social media. Social media is a platform with short video content that contains entertainment, information, and education [3]. According to the We Are Social survey in 2023, Indonesia is in second place with the most significant number of social media users worldwide after the United States. Social media users in Indonesia reached 106.52 million in October 2023. Despite its many benefits, the platform also raises concerns regarding the potential negative impact of addiction.

Addiction is fun and done excessively, making people unable to control themselves [4]. Addiction is a medical and psychiatric condition where a person uses something beyond the limit and can hurt his life if done excessively and repeatedly [5]. Addiction does not only occur due to dependence on addictive substances; some activities or behaviours can also cause addiction, such as internet use [6]. Excessive use of social media can cause several negative impacts, such as insomnia, decreased productivity, difficulty managing time, and health problems. In addition, social media addiction can also lead to mental health disorders such as anxiety, depression, and even Tourette's Syndrome. Tourette's syndrome is a disorder that usually appears in childhood and often continues into adulthood. People with this disease tend to make sudden and repetitive movements or imitate sounds under their conscious awareness [7]. The severe stage of addiction is related to Tourette's Syndrome with the type of MSMI-FTB (Mass Social Media Illness Functional Tourette-like Behavior). MSMI-FTB is the result of excessive social media use that can lead to compulsive or impulsive behaviour patterns, similar to some of the symptoms of Tourette Syndrome [8]. In this case, it is essential to control and treat. Advertising and education about the negative impact of social media can be one of the control measures. Other strategies can include medication and seeking medical help if the symptoms of addiction begin to interfere with daily activities [9].

Mathematical modelling is essential in understanding, predicting, and controlling the spread of infectious diseases, addiction, and other problems. The mathematical model used is the modified SEA_1A_2RQ model, which has six sub-populations with the following assumptions: Susceptible (S) is a population of individuals who do not have social media but are susceptible to addiction due to environmental factors. Exposed (E) is a population of individuals who have social media but have not reached the level of addiction. Addicted$_1$ (A_1) is a population of individuals who are addicted to social media in a mild stage; here, users begin to be unable to divide their time. Addicted$_2$ (A_2) is a population of individuals who are addicted to social media in a severe stage; here, users experience Tourette Syndrome. Recovery (R) is a population of individuals who are recovering with medication from social media addiction. Quit (Q) is a population of individuals who finally stop (uninstall) using social media [10]. The SEA_1A_2RQ model is specifically designed to capture the complexity of addiction dynamics, particularly in the context of social media platforms, which have unique and potent psychological impacts on users. This model is an extension of classic epidemic models, modified to reflect not only the viral nature of social media addiction but also the progression through different stages from susceptibility to recovery or quitting. A model like SEA_1A_2RQ is essential because it allows us to examine how different interventions affect the population at each stage, providing a granular view of addiction dynamics. The control variables, u_1 and u_2, are integrated to evaluate preventive measures, such as education and awareness (social campaigns to reduce initial exposure to social media), and treatment methods aimed at addicted individuals, such as therapy and digital detox strategies. Without such a model, it would be difficult to systematically assess the impact of these controls and determine the most effective strategy for reducing addiction rates.

Additionally, this model incorporates assumptions about the social spread of addictive behaviour, recognizing that exposure to addicted individuals can increase the likelihood of addiction in susceptible individuals. This makes it especially relevant for social media addiction, where peer influence and viral content play a critical role. By simulating different scenarios and control strategies, the model provides valuable insights into how addiction can be controlled at the population level and within specific subgroups.

This study will analyze the optimal control problem in the social media addiction model by using two control variables, namely control (u_1), which can be in the form of education in the form of socialization about the negative impact of social media and control in the form of appropriate treatment (u_2) to restore addiction such as deactivation of internet connections, habituation of discipline in the use of social media and for severe stage addiction can be given therapy and administration of antipsychotic drugs. The control variables (u_1) and (u_2) are defined in the base model so that the model formulation with controls will be obtained. Furthermore, the Pontryagin Maximum Principle is used to get the optimal control solution so that the number of individuals addicted to social media can be reduced. From the numerical simulation results, the models that have not been given control and those that have been given control will be compared so that the differences in each sub-population will be known when given control u_1 and u_2 and without being given control.

The urgency of analyzing social media addiction stems from its increasing prevalence, particularly among younger populations, and its significant impact on mental health, productivity, and social behaviour. By understanding the stages of addiction and the effect of different control measures, this research aims to provide insights into how to mitigate the spread of addiction and assist individuals in recovery. The SEA_1A_2RQ model simulates various intervention strategies, allowing policymakers and healthcare professionals to implement more effective prevention and treatment programs. The outcomes of this analysis are expected to contribute directly to efforts in curbing social media addiction, a critical issue in the digital age.

2. METHODS

The research methodology for calculating the optimal control of the SEA_1A_2RQ model to mitigate social media addiction involves the following steps:

1. Model formulation with control: develop the mathematical model for social media addiction incorporating control variables. The model should describe the transitions between Susceptible (S), Exposed (E), Addicted (A_1), Recovery (R), and Quit (Q) states, along with the impact of control (u_1) and treatment (u_2)) variables.
2. Objective function: define the objective function to minimize the number of addicted individuals and the cost of implementing control measures. The objective function should balance reducing addiction and the resources expended.
3. Hamiltonian function: formulate the Hamiltonian function by incorporating the state, control, and Lagrange multipliers (co-state variables). The Hamiltonian will be used to derive the necessary conditions for optimal control.
4. State and co-state equations: derive the differential equations for the state variables based on the SEA_1A_2RQ model. Simultaneously, the co-state (adjoint) equations from the Hamiltonian function are derived to describe the dynamics of the co-state variables.
5. Stationary conditions and optimal control values: determine the stationary conditions by setting

the partial derivatives of the Hamiltonian with respect to the control variables to zero. Solve these conditions to find the expressions for the optimal control variables (u_1) and (u_2).

6. Numerical simulation: implement a numerical simulation of the SEA_1A_2RQ model using the 4th-order Runge-Kutta method. This step involves solving the state and co-state equations over time to observe the system's behavior with and without optimal control.
7. Comparison and analysis: analyze the simulation results to compare the populations of addicted individuals in scenarios with and without control. Assess the effectiveness of the optimal control strategy by evaluating the reduction in addiction levels over a specified period.

By following these steps, this research aims to develop and validate an optimal control strategy that effectively reduces social media addiction, providing valuable insights into managing and mitigating this modern behavioral issue.

3. RESULTS

3.1. The Compartment Diagrams based on References

The mathematical model of social media addiction, incorporating control variables u_1 and u_2, can be described in Figure 1. In this figure, the placement of the controls u_1 and u_2 in the SEA_1A_2RQ model is crucial for addressing social media addiction. Control u_1 is positioned along the path between the Susceptible (S) and Exposed (E) populations. This control represents preventive actions, such as education and awareness campaigns about the adverse effects of social media, aimed at reducing the number of individuals transitioning from Susceptible to Exposed. By implementing u_1, the model assumes that fewer individuals will begin using social media due to preventive measures. On the other hand, control u_2 is placed along the paths leading from the Addicted₁ (A_1) and Addicted₂ (A_2) populations to the Recovery (R) population. This control focuses on treatment and therapeutic interventions, such as therapy and medication, to help addicted individuals recover. The assumption is that u_2 will reduce the number of addicted individuals by encouraging recovery. The model assumes that adequate controls can reduce addiction rates and promote recovery, but it also recognizes that relapses may occur depending on the effectiveness of treatment. These controls work together to reduce the overall population affected by social media addiction, emphasizing both prevention and recovery efforts.

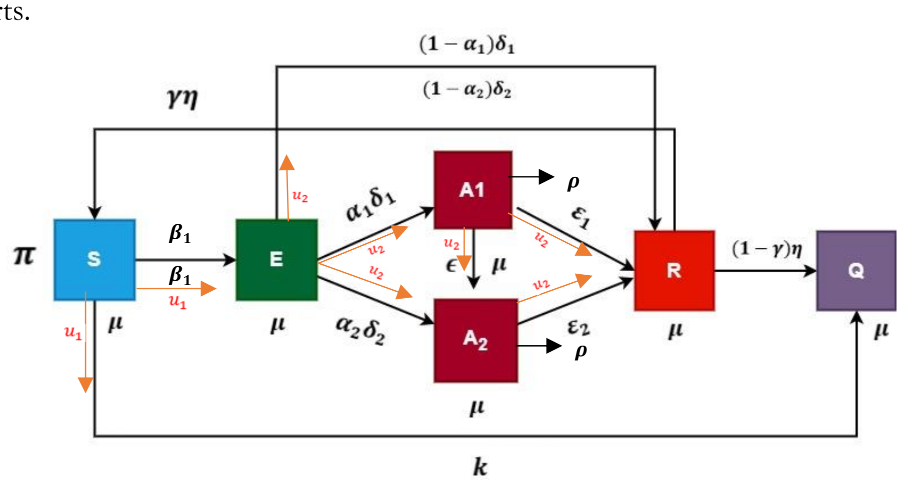

Figure 1. SEA_1A_2RQ model with u_1 and u_2 controls

3.2. Variables and Parameter Values Associated with Compartment Diagrams

This subsection outlines the variables and parameter values that are integral to the compartment diagrams in Figure 1, providing a detailed description of each element and its role in the model.

Table 1. Variable description

Variable	Description	Initial Condition	Source
$S(t)$	Total population of vulnerable individuals who do not use social media	100	[9]
$E(t)$	Population size of social media users	10	[9]
$A_1(t)$	Population size of mildly addicted individuals	50	[9]
$A_2(t)$	Population size of severely addicted individuals	50	[9]
$R(t)$	Population size of individuals in treatment	0	[9]
$Q(t)$	Population size of individuals who stopped using social media	100	[9]

Table 2. SEA_1A_2RQ model parameter description

Parameter	Description	Value	Unit	Source
π	The recruitment rate of Susceptible individuals	0.5	$\frac{1}{year}$	[9]
μ	Natural Date Rate	0.25	$\frac{1}{year}$	[11]
β_1	Transmission rate of social media use from mildly addicted individuals to Susceptible individuals due to surrounding influences	0.6	$\frac{1}{year}$	[11]
β_2	Transmission rate of social media use from severely addicted individuals to Susceptible individuals due to surrounding influences	0.58	$\frac{1}{year}$	[12]
σ_1	Contact with susceptible individuals and mildly addicted individuals	0.5	$\frac{1}{year}$	[11]
σ_2	Contacts of susceptible individuals and severely addicted individuals	0.22	$\frac{1}{year}$	[12]
k	Rate of exposed individuals who do not use social media	0.01	$\frac{1}{year}$	[9]
α_1	The proportion of exposed individuals with mild addiction	0.71		[13]
α_2	The proportion of exposed individuals who are severely addicted	0.22		[14]
δ_1	Individuals who dropped out of the exposed population	0.25	$\frac{1}{year}$	[13]
δ_2	Individuals who dropped out of the exposed population	0.21	$\frac{1}{year}$	[12]
ϵ	Rate of movement of addicted individuals from mild to severe stage	0.688	$\frac{1}{year}$	[14]

Parameter	Description	Value	Unit	Source
ε_1	Movement rate of mildly addicted individuals entering treatment	0.7	$\frac{1}{year}$	[11]
ε_2	Movement rate of severely addicted individuals entering treatment	0.001	$\frac{1}{year}$	[15]
γ	The proportion of individuals who have entered treatment but are still vulnerable to addiction	0.35		[13]
η	Individuals who left the treatment class	0.4	$\frac{1}{year}$	[11]
ρ	The death rate due to social media addiction	0.01	$\frac{1}{year}$	[9]

3.3. Differential Equations According to Compartment Diagram

To minimize social media addiction, it is necessary to formulate an optimal control problem with two control variables, $u_1(t)$ and $u_2(t)$. The control variable $u_1(t)$ represents efforts aimed at preventing vulnerable populations from coming into contact with addicts through socialization and education to the public regarding the negative impact of social media. The control variable $u_2(t)$ is used to control for addicted individuals to be given treatment measures to recover from addiction. The following system of differential equations gives the mathematical model of controlled social media addiction:

$$\frac{dS}{dt} = \pi + \gamma\eta R - (1-u_1)(\beta_1\sigma_1 A_1 + \beta_2\sigma_2 A_2)S - (k + u_1 + \mu)S,$$
$$\frac{dE}{dt} = (1-u_1)(\beta_1\sigma_1 A_1 + \beta_2\sigma_2 A_2)S - (\delta_1 + \delta_2 + u_2 + \mu)E,$$
$$\frac{dA_1}{dt} = \alpha_1(\delta_1 + u_2)E - [\epsilon + u_2 + \varepsilon_1 + \rho + \mu]A_1,$$
$$\frac{dA_2}{dt} = \alpha_2(\delta_2 + u_2)E + (\epsilon + u_2)A_1 - [\varepsilon_2 + u_2 + \rho + \mu]A_2,$$
$$\frac{dR}{dt} = [(1-\alpha_1)(\delta_1 + u_2) + (1-\alpha_2)(\delta_2 + u_2)]E + (\varepsilon_1 + u_2)A_1 + (\varepsilon_2 + u_2)A_2 - (\eta + \mu)R,$$
$$\frac{dQ}{dt} = (k + u_1)S + (1-\gamma)\eta R - \mu Q.$$

3.4. The Objective Function

The objective function is formulated as follows:

$$J = \int_0^{tf} \left(b_1 E + b_2 A_1 + b_3 A_2 + \frac{1}{2}(w_1 u_1^2 + w_2 u_2^2) \right) dt,$$

where $\boldsymbol{t_0}$ is the initial state, $\boldsymbol{t_f}$ is the final state, $\boldsymbol{b}$ is a constant weight, and $\boldsymbol{w}$ is coefficient.

3.5. The Optimal Solution using Pontryagin's Maximum Principle

The optimal solution to the optimal control problem can be obtained from the Hamiltonian function formulated as follows:

$$\mathcal{H} = b_1 E + b_2 A_1 + b_3 A_2 + \frac{1}{2} w_1 u_1^2 + \frac{1}{2} w_2 u_2^2$$
$$+\lambda_1[0{,}5 + (0{,}35)(0{,}4)R - (1-u_1)((0{,}6)(0{,}5)A_1 + (0{,}58)(0{,}22)A_2)S - (0{,}01 + u_1 + 0{,}25)S]$$

$$+\lambda_2[(1-u_1)((0{,}6)(0{,}5)A_1+(0{,}58)(0{,}22)A_2)S-(0{,}25+0{,}21+u_2+0{,}25)E]$$
$$+\lambda_3[0{,}71(0{,}25+u_2)E-(0{,}688+u_2+0{,}7+0{,}01+0{,}25)A_1\,]$$
$$+\lambda_4[0{,}22(0{,}21+u_2)E+(0{,}688+u_2)A_1-(0{,}001+u_2+0{,}01+0{,}25)A_2]$$
$$+\lambda_5[[(1-0{,}71)(0{,}25+u_2)+(1-0{,}22)(0{,}21+u_2)]E+(0{,}7+u_2)A_1+(0{,}001+u_2)A_2$$
$$-(0{,}4+0{,}25)R]+\lambda_6[(0{,}01+u_1)S+(1-0{,}35)0{,}4R-0{,}25Q],$$

with $\lambda_i, i=1,\dots,6$ are the adjoint function variables to be determined. The state and co-state equations will be determined based on the Hamiltonian above. The state equation is obtained as follows:

$$\frac{\partial\mathcal{H}}{\partial\lambda_1}=0{,}5+(0{,}35)(0{,}4)R-(1-u_1)((0{,}6)(0{,}5)A_1+(0{,}58)(0{,}22)A_2)S-(0{,}01+u_1+0{,}25)S,$$
$$\frac{\partial\mathcal{H}}{\partial\lambda_2}=(1-u_1)((0{,}6)(0{,}5)A_1+(0{,}58)(0{,}22)A_2)S-(0{,}25+0{,}21+u_2+0{,}25)E,$$
$$\frac{\partial\mathcal{H}}{\partial\lambda_3}=0{,}71(0{,}25+u_2)E-(0{,}688+u_2+0{,}7+0{,}01+0{,}25)A_1,$$
$$\frac{\partial\mathcal{H}}{\partial\lambda_4}=0{,}22(0{,}21+u_2)E+(0{,}688+u_2)A_1-(0{,}001+u_2+0{,}01+0{,}25)A_2,$$
$$\frac{\partial\mathcal{H}}{\partial\lambda_5}=\;[(1-0{,}71)(0{,}25+u_2)+(1-0{,}22)(0{,}21+u_2)]E+(0{,}7+u_2)A_1$$
$$+(0{,}001+u_2)A_2-(0{,}4+0{,}25)R,$$

$$\frac{\partial\mathcal{H}}{\partial\lambda_6}=(0{,}01+u_1)S+(1-0{,}35)0{,}4R-0{,}25Q,$$

with initial conditions $S(0)=S_0, E(0)=E_0, A_1(0)=A_{1\,0}, A_2(0)=A_{2\,0}, R(0)=R_0, Q(0)=Q_0$.

There is an adjoint variable $\lambda_i, i=1,\dots,6$ that satisfies the equation below:

$$\frac{d\lambda_1}{dt}=-\frac{\partial\mathcal{H}}{\partial S}(t)$$

$$\frac{d\lambda_2}{dt}=-\frac{\partial\mathcal{H}}{\partial E}(t)$$

$$\frac{d\lambda_3}{dt}=-\frac{\partial\mathcal{H}}{\partial A_1}(t)$$

$$\frac{d\lambda_4}{dt}=-\frac{\partial\mathcal{H}}{\partial A_2}(t)$$

$$\frac{d\lambda_5}{dt}=-\frac{\partial\mathcal{H}}{\partial R}(t)$$

$$\frac{d\lambda_6}{dt}=-\frac{\partial\mathcal{H}}{\partial Q}(t)$$

The co-state equation is obtained as follows:

$$\frac{d\lambda_1}{dt} = -\frac{\partial\mathcal{H}}{\partial S} = \lambda_1\big((1-u_1)((0{,}6)(0{,}5)A_1 + (0{,}58)(0{,}22)A_2)S + 0{,}01 + (1-u_1) + 0{,}25\big)$$
$$- \lambda_2\big((1-u_1)((0{,}6)(0{,}5)A_1 + (0{,}58)(0{,}22)A_2)S\big) - \lambda_6\,(0{,}01 + u_1)$$

$$\frac{d\lambda_2}{dt} = -\frac{\partial\mathcal{H}}{\partial E} = -b_1 + \lambda_2(0{,}25 + 0{,}21 + u_2 + 0{,}25) - \lambda_3\big(0{,}71(0{,}25 + u_2)\big)$$
$$- \lambda_4\big(0{,}22(0{,}21 + u_2)\big) - \lambda_5\big((1-0{,}71)(0{,}25 + u_2) + (1-0{,}22)(0{,}21 + u_2)\big)$$

$$\frac{d\lambda_3}{dt} = -\frac{\partial\mathcal{H}}{\partial A_1} = -b_2 + \lambda_1\big((1-u_1)(0{,}6)(0{,}25)S\big) - \lambda_2\big((1-u_1)(0{,}6)(0{,}5)S\big)$$
$$- \lambda_3(0{,}688 + u_2 + 0{,}7 + 0{,}01 + 0{,}025) - \lambda_4(0{,}688 - u_2) - \lambda_5(0.001 + u_2)$$

$$\frac{d\lambda_4}{dt} = -\frac{\partial\mathcal{H}}{\partial A_2} = -b_3 + \lambda_1\big((1-u_1)(0{,}58)(0{,}22)S\big) - \lambda_2\big((1-u_1)(0{,}58)(0{,}22)S\big)$$
$$+ \lambda_4\big(0{,}001 + u_{2q} + 0{,}01 + 0{,}25\big) - \lambda_5(0.001 + u_2)$$

$$\frac{d\lambda_5}{dt} = -\frac{\partial\mathcal{H}}{\partial R} = -\lambda_1((0{,}35)(0{,}4)) + \lambda_5(0{,}4 + 0{,}25) - \lambda_6\big((1-0{,}35)4\big)$$

$$\frac{d\lambda_6}{dt} = -\frac{\partial\mathcal{H}}{\partial Q} = \lambda_6(0.25)$$

Then, the values of u_1 and u_2 are sought so that the optimal condition is that the derivative of the Hamiltonian function must be equal to 0.

$$\frac{\partial\mathcal{H}}{\partial u} = 0 \iff \frac{\partial\mathcal{H}}{\partial u_1} = 0 \text{ dan } \frac{\partial\mathcal{H}}{\partial u_2} = 0.$$

If $u_{1,2}(t) = 1$, then the control can be considered optimal or effective, and if $u_{1,2}(t) = 0$, then it can be said that the provision of control has no effect in reducing the number of Exposed, Addicted$_1$ and Addicted$_2$. With transversality condition, $\lambda_i(tf) = 0, i = 1, \dots, 6$ and optimal control variable u_1^* and u_2^* are functions given by:

$$u_1^* = \max\left\{0, min\left(1, \frac{(\lambda_2-\lambda_1)(0{,}6)(0{,}5)A_1S+(0{,}58)(0{,}22)A_2S+(\lambda_1-\lambda_6)S}{w_1}\right)\right\},$$
$$u_2^* = \max\left\{0, min\left(1, \frac{(\lambda_2-\lambda_5)E+(\lambda_5-\lambda_3)0{,}71E-A_1+(\lambda_5-\lambda_4)0{,}22E-A_2-(\lambda_4)A_1}{w_2}\right)\right\}.$$

3.6. Numerical Simulation

Numerical simulations will be performed using the 4th-order Runge-Kutta method to evaluate the effectiveness of community control and treatment strategies for social media addiction. These simulations will be carried out using Google Colab. Table 1 presents the initial values, while Table 2 provides the parameter values used in the model. The simulation is conducted over a time interval from $t_0 = 0$ to $t_f = 4$ years. The weighting constants for the Exposed, Addicted1, and Addicted2 populations are $b_1 = 1$, $b_2 = 2$, and $b_3 = 2$, respectively. Additionally, the weights for control and treatment costs are $w_1 = 10$ and $w_2 = 10$. The results from these numerical simulations, which focus on optimal control of the Susceptible, Exposed, Addicted1, Addicted2, Recovery, and Quit (SEA_1A_2RQ) model with controls u_1 and u_2, are as follows:

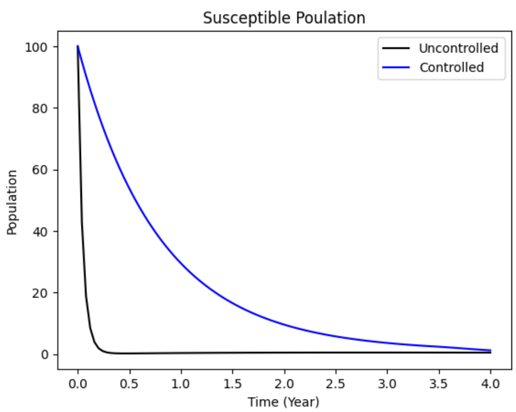

Figure 2. Simulation of $S(t)$

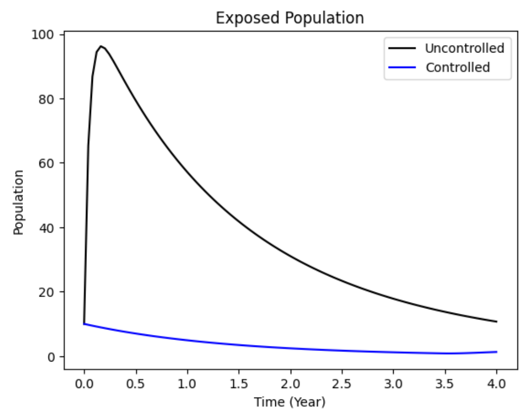

Figure 3. Simulation of $E(t)$

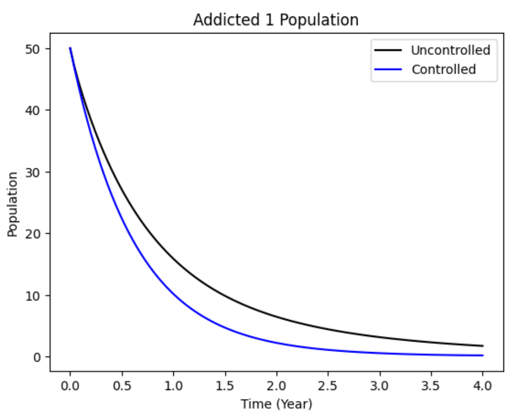

Figure 4. Simulation of $A_1(t)$

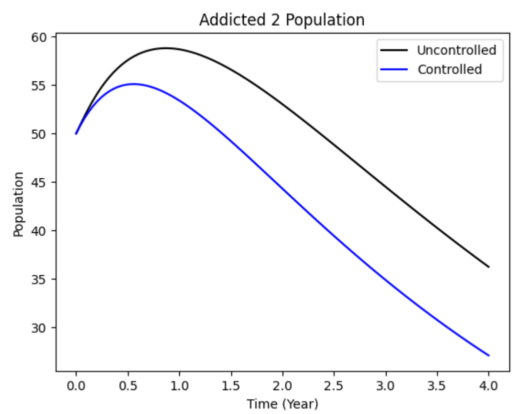

Figure 5. Simulation of $A_2(t)$

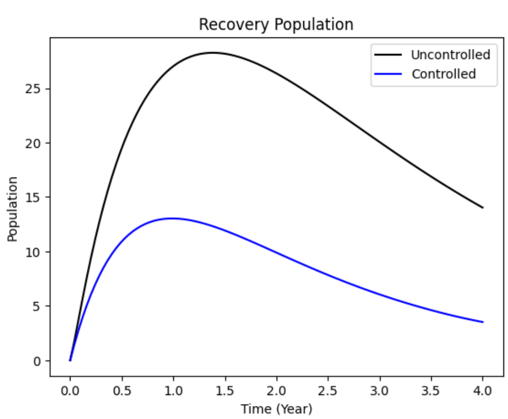

Figure 6. Simulation of $R(t)$

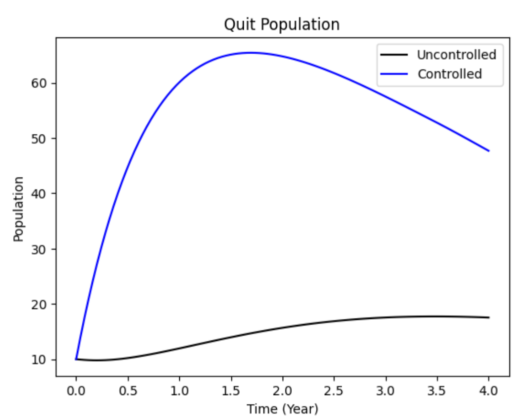

Figure 7. Simulation of $Q(t)$

Figures 2 through 7 in the simulation of the modified social media addiction model clearly demonstrate the effectiveness of control and treatment strategies in reducing both susceptible and exposed populations. Figure 2 illustrates the decline of the susceptible population (S) over time, with a more pronounced reduction under the application of optimal control (u_1). This highlights the success of education and awareness campaigns in preventing individuals from initially engaging with social media. Figure 3 displays the dynamics of the exposed population (E), representing individuals who have used social media but are not yet addicted. The graph shows a sharper decrease when both control (u_1) and treatment (u_2) are applied, underscoring the effectiveness of these interventions in preventing progression from exposure to addiction. Figure 4 shows the mildly addicted population (A_1), which experiences a significant reduction following the implementation of control measures. This suggests that early interventions can effectively curb the spread of mild addiction. Figure 5 reflects the dynamics of the severely addicted population (A_2), where the decline is slower compared to the mildly addicted group. Nevertheless, the graph still demonstrates the positive impact of treatment (u_2) in reducing severe addiction levels. Figure 6 shows an increasing trend in the recovery population (R) as a result of effective treatment and moderated social media use, further supported by the control measures (u_2). Figure 7 highlights the rising number of individuals who have quit using social media (Q), confirming the efficacy of both control strategies in helping individuals overcome addiction. Overall, each sub-population in the model shows clear distinctions between controlled and uncontrolled scenarios, confirming that a combination of social interventions and treatment significantly reduces social media addiction. These results emphasize the importance of both preventive measures and recovery support in addressing the addictive behaviors associated with social media.

4. DISCUSSION

The results of this study demonstrate the effectiveness of applying optimal control to reduce social media addiction through preventive measures and treatment strategies. By incorporating the control variables u_1 (preventive education) and u_2 (treatment), the numerical simulations reveal significant reductions in the number of individuals in both the mildly and severely addicted populations. Specifically, the implementation of $u1$ successfully lowers the transition from susceptible to exposed individuals, while u_2 accelerates the recovery process in addicted individuals.

Comparison with Related Studies

The findings align with previous research on addiction dynamics in digital platforms, particularly studies that use mathematical models to analyze behavioral addictions. For example, [9] also used optimal control in their study on social media addiction, concluding that educational interventions and treatment could significantly reduce the addicted population. Our results confirm and extend these findings by incorporating a more detailed compartmental model that distinguishes between different levels of addiction severity, specifically the mild (A_1) and severe (A_2) stages of addiction, as well as the inclusion of relapse rates. Similarly, [13] explored optimal control strategies in the context of online game addiction, which shares many behavioral similarities with social media addiction. Their research found that combining awareness campaigns and strict regulation could mitigate addiction over time. In comparison, our model offers a more specific focus on social media platforms, where peer influence and viral content are major contributing factors to addiction dynamics. The addition of u_1 as a preventive measure targeting susceptible individuals emphasizes the importance of early interventions to curb the spread of addictive behaviors, which is also echoed in their findings.

Moreover, Paulus et al. [12] examined the influence of social media-induced behaviors on mental health, particularly in adolescents. Their study highlights the psychological impact of excessive social media usage, linking it to conditions such as Tourette-like behaviors. This aligns with our model, which assumes a progression from mild to severe addiction stages, with severe cases potentially leading to more pronounced mental health disorders. The inclusion of relapse and treatment dynamics in our study supports the idea that ongoing intervention is crucial to preventing the recurrence of severe symptoms, a point also stressed in Paulus' findings.

Practical Implications

The practical implications of this research are relevant for policymakers and health organizations looking to combat social media addiction. Preventive campaigns (represented by u_1) should focus on educating vulnerable populations, particularly younger individuals, about the risks associated with excessive social media use. On the other hand, treatment strategies (represented by u_2) could involve therapy and controlled social media usage regimens for those already showing signs of addiction.

Compared to existing models, this study provides a more detailed approach by incorporating different stages of addiction severity and considering both prevention and treatment simultaneously. These distinctions are critical for formulating targeted intervention strategies. For instance, while previous models treated addiction as a single-stage phenomenon, this study highlights the need for differentiated responses based on the individual's addiction stage (A_1 vs. A_2).

Limitations and Future Research

Although the model presents valuable insights into social media addiction, it has certain limitations. One major limitation is the assumption of homogeneity within each population group. In reality, individual behavior can vary significantly based on factors such as age, social environment, and psychological predispositions, which are not fully captured in the current model. Future research could address this by incorporating more complex, agent-based models or by stratifying the population based on these variables.

Another limitation is the simplified assumption of constant control efforts over time. In real-world applications, preventive campaigns and treatment measures may fluctuate based on factors such as budget constraints, public interest, or technological advancements. Future studies should explore models where control variables change dynamically over time, adapting to the evolving nature of social media platforms and addiction patterns.

Lastly, this study does not fully consider the potential for cross-platform addiction. Many individuals do not limit their usage to a single social media platform but are engaged across multiple platforms. Incorporating multi-platform addiction models could provide a more comprehensive view of the behavioral patterns driving social media addiction and its optimal control.

4. CONCLUSIONS

This research successfully demonstrates the application of optimal control strategies within the (SEA_1A_2RQ) model to mitigate social media addiction. By employing Pontryagin's maximum principle and integrating time-dependent control variables for preventive education and treatment, the study reveals significant reductions in the number of individuals addicted to social media. Numerical simulations using the 4th-order Runge-Kutta method illustrate the effectiveness of these interventions, showing marked differences between controlled and uncontrolled scenarios over a four-year period. The results affirm that a combination of control (u_1) and treatment (u_2) measures can substantially decrease addiction rates, providing a viable framework for addressing the pervasive issue of social media addiction. This research offers valuable insights for policymakers and healthcare providers in developing comprehensive strategies to combat social media addiction and improve public mental health.

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