ISSN: 2301-4490

THE 5th INTERNATIONAL CONFERENCE ON GREEN TECHNOLOGY

"Science and Technology in Islamic Perspective: Synergy and Theirs Contribution to Empower Nations"

5th GREEN TECH INTERNATIONAL CONFERENCE

PROCEEDING

Issue: 5 / 2015

7th-8th NOVEMBER 2014
Soekarno Hatta Building, 5th floor
Maulana Malik Ibrahim State Islamic University of Malang

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Proceeding of The 5th International Conference on Green Technology; ISSN: 2301-4490; Issue: 5/2015; Accepted: November, 2014; Published: 2015

An Application of α-Cut Fuzzy $\tilde{\bar{\bar{X}}} - \tilde{S}$ Control Charts Using Fuzzy Trapezoidal Number

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Keywords: fuzzy trapezoidal number, $\tilde{\bar{\bar{X}}} - \tilde{S}$ control charts, industrialized building,

Abstract. Statistical Process Control (SPC) is a collection of statistical and analytical tools that can be used to achieve process stability and variability reduction about the process target value. The most important tool in SPC is the control chart which was first introduced by Shewhart in the 1920's. There are two familiar control chart techniques called the traditional variable control charts. There are $\bar{X} - R$ and $\bar{X} - S$. These consists of three lines namely Center Line (CL), Upper Control Limit (UCL), and Lower Control Limit (LCL). These limits are represented by single numerical values. For many problems, control limits could not be so precise because of the measurement system including operator, gauges, and environmental conditions. Fuzzy set theory is a useful tool to handle this problem because the result is more accurate and flexible evaluation. Numeric control limits can be transformed to fuzzy control limits by using membership functions on fuzzy set. The fuzzy α–level fuzzy midrange $\tilde{\bar{\bar{X}}} - \tilde{R}$ and $\tilde{\bar{\bar{X}}} - \tilde{S}$ control charts using fuzzy trapezoidal number have been constructed by Pandurangan and Varadharajan [4]. In this paper, an application of fuzzy $\tilde{\bar{\bar{X}}} - \tilde{S}$ control charts using trapezoidal fuzzy number on controlling piston inner diameter in compressors is presented.

Introduction

The control chart originated in the early 1920s, it has become a powerful tool in Statistical Process Control (SPC) that is the most widely used in industrial processes. Control charts are designed to monitor the process of change in mean and variance, they also reflect the ability of the process. Control charts have two types: variable and attribute control charts [2]. Techniques of statistical process control are widely used by the manufacturing industry to detect and eliminate defects during production. Control chart includes three main lines which are the centre line (CL), the lower control limit (LCL) and the upper control limit (UCL). The process is considered to be in-control if the sample point lies between the LCL and the UCL. Otherwise, if the sample point lies outside the control limits can be regarded as evidence representing that the process out of control [3].

Many studies were done to combine statistical methods and fuzzy set theory. Fuzzy sets theory was first introduced by Zadeh [7]. The fuzzy set theory is a more suitable tool for handling attribute data since these data may be expressed in linguistic terms such as “very good”, “good”, “medium”, “bad”, and “very bad”. There are few papers on fuzzy variable control chart and their application. For example, Rowlands and Wang [5] introduced fuzzy-SPC methods based on the application of fuzzy logic to the SPC-zone rules. El-Shal and Morris [1] modified SPC-zone rules to reduce false alarms and detect the real faults. Zarandi et al. [8] presented a new hybrid method based on a combination of fuzzified sensitivity criteria and fuzzy adaptive sampling rules to determine the sample size and sample interval of the control charts. Senturk and Erginel [6] developed the fuzzy $\tilde{\bar{\bar{X}}} - \tilde{R}$ and $\tilde{\bar{\bar{X}}} - \tilde{S}$ control chart with α-cuts by using α-level fuzzy midrange based triangular fuzzy number and their application. Whereas, Pandurangan and Varadharajan [4] constructed the fuzzy $\tilde{\bar{\bar{X}}} - \tilde{R}$ and $\tilde{\bar{\bar{X}}} - \tilde{S}$ control chart with α-cuts by using α-level fuzzy midrange based trapezoidal fuzzy number and the application of fuzzy $\tilde{\bar{\bar{X}}} - \tilde{R}$ control chart. Based on the paper of Pandurangan and Varadharajan, has applied fuzzy $\tilde{\bar{\bar{X}}} - \tilde{R}$ control chart, the purpose of this study to apply α-cut fuzzy $\tilde{\bar{\bar{X}}} - \tilde{S}$ control chart using α-level fuzzy midrange based trapezoidal fuzzy number on controlling piston inner diameter in compressors.

α-Level Fuzzy Midrange for α-Cut Fuzzy $\tilde{\bar{\bar{X}}}$ Control Chart Based on Standard Deviation

The control limits and centre line for α-cut fuzzy $\tilde{\bar{\bar{X}}}$ control chart based on standard deviation using α-level fuzzy midrange are

$$UCL^{\alpha}_{mr-\bar{X}} = CL^{\alpha}_{mr-\bar{X}} + A_3\left(\frac{\bar{S}^{\alpha}_a + \bar{S}^{\alpha}_d}{2}\right)$$

$$CL^{\alpha}_{mr-\bar{X}} = f^{\alpha}_{mr-\bar{X}}(\tilde{C}L) = \frac{\bar{\bar{X}}^{\alpha}_a + \bar{\bar{X}}^{\alpha}_d}{2}$$

$$LCL^{\alpha}_{mr-\bar{X}} = CL^{\alpha}_{mr-\bar{X}} - A_3\left(\frac{\bar{S}^{\alpha}_a + \bar{S}^{\alpha}_d}{2}\right)$$

where,

$$\bar{\bar{X}}^{\alpha}_a = \bar{\bar{X}}_a + \alpha(\bar{\bar{X}}_b - \bar{\bar{X}}_a);\ \ \bar{\bar{X}}^{\alpha}_d = \bar{\bar{X}}_d + \alpha(\bar{\bar{X}}_d - \bar{\bar{X}}_c)$$

$$\bar{S}^{\alpha}_a = \bar{S}_a + \alpha(\bar{S}_b - \bar{S}_a);\ \bar{S}^{\alpha}_d = \bar{S}_d + \alpha(\bar{S}_d - \bar{S}_c)$$

The definition of α-level fuzzy midrange of sample j, j = 1, 2, ..., m for fuzzy $\tilde{\bar{\bar{X}}}$ control chart is

$$S^{\alpha}_{mr-\bar{X}.j} = \frac{(\bar{X}_{aj} + \bar{X}_{dj}) + \alpha[(\bar{X}_{bj} - \bar{X}_{aj}) - (\bar{X}_{dj} - \bar{X}_{cj})]}{2},\ \ j = 1,2,\dots,m$$

Then, the condition of process control for each sample can be defined as:

$$\text{Process control} = \begin{cases} \text{in control} & ;\ \text{for } LCL^{\alpha}_{mr-\bar{X}} \le S^{\alpha}_{mr-\bar{X}.j} \le UCL^{\alpha}_{mr-\bar{X}} \\ \text{out of control} & ;\ \text{otherwise} \end{cases}$$

α-Level Fuzzy Midrange for α-Cut Fuzzy $\tilde{S}$ Control Chart

The control limits of α-level fuzzy midrange for α-cut Fuzzy $\tilde{S}$ control chart based on trapezoidal fuzzy number can be calculated as follows

$$UCL^{\alpha}_{mr-S} = B_4 f^{\alpha}_{mr-S}(\widetilde{CL})$$

$$CL^{\alpha}_{mr-S} = f^{\alpha}_{mr-S}(\widetilde{CL}) = \frac{\bar{S}^{\alpha}_a + \bar{S}^{\alpha}_d}{2}$$

$$LCL^{\alpha}_{mr-S} = B_3 f^{\alpha}_{mr-S}(\widetilde{CL})$$

The definition of α-level fuzzy midrange of sample j, j = 1, 2, ..., m for fuzzy $\tilde{S}$ control chart can be calculated as follows

$$S^{\alpha}_{mr-S.j} = \frac{(S_{aj} + S_{dj}) + \alpha[(S_{bj} - S_{aj}) - (S_{dj} - S_{cj})]}{2}$$

Then, the condition of process control for each sample can be defined as:

$$\text{Process control} = \begin{cases} \text{in control} & ;\ \text{for } LCL^{\alpha}_{mr-S} \le S^{\alpha}_{mr-S.j} \le UCL^{\alpha}_{mr-S} \\ \text{out of control} & ;\ \text{otherwise} \end{cases}$$

Methodology

An application of was given by Pandurangan and Varadharajan (2011) using trapezoidal fuzzy numbers on controlling piston inner diameters in compressors. The same data have been considered with the first fifteen samples each with size 5 (the total measurements is 5x15 = 75). These measurements are converted into trapezoidal fuzzy numbers and given in Table 1. Fuzzy control limits are calculated according to the procedures given in the previous sections.

The following procedures are used to apply the fuzzy $\tilde{\bar{\bar{X}}} - \tilde{S}$ control charts.

a. Calculate the α level fuzzy midrange for fuzzy $\tilde{\bar{\bar{X}}} - \tilde{S}$ control charts by using α level fuzzy midrange transformation techniques.
b. Apply $\tilde{\bar{\bar{X}}} - \tilde{S}$ control charts by using the numerical example. Classify the condition of process control for each sample.

Results and Discussion

An Application of α–cut Fuzzy $\tilde{\bar{\bar{X}}} - \tilde{S}$ Control Charts

For $n = 5$, $A_3 = 1.427$, $B_3 = 0$ and $B_4 = 2.089$. Where A_3, B_3 and B_4 is obtained from the coefficients table for variable control charts.

Table 1. The Fuzzy Measurement Values using Trapezoidal Fuzzy Number

Sample No	*X*a					*X*b					*X*c					*X*d				
	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
1	5.71	5.50	5.43	5.20	5.51	5.73	5.57	5.45	5.25	5.53	5.75	5.60	5.46	5.27	5.55	5.76	5.62	5.47	5.28	5.56
2	5.41	5.52	5.25	5.51	5.65	5.43	5.57	5.29	5.53	5.69	5.44	5.58	5.3	5.56	5.7	5.45	5.69	5.31	5.58	5.71
3	5.25	5.51	5.00	5.20	5.31	5.29	5.53	5.13	5.25	5.33	5.32	5.54	5.17	5.28	5.37	5.33	5.55	5.19	5.29	5.39
15	5.62	5.48	5.42	5.18	5.41	5.64	5.55	5.46	5.23	5.46	5.68	5.59	5.49	5.26	5.49	5.73	5.63	5.54	5.31	5.53

α-Level Fuzzy Midrange for α - Cut $\tilde{\bar{\bar{X}}}$ Control Chart Based on Standard Deviation

The values for $\tilde{\bar{S}}$ and $\bar{\bar{X}}$, is given below

$$\tilde{\bar{S}} = \left(\tilde{\bar{S}}_a, \tilde{\bar{S}}_b, \tilde{\bar{S}}_c, \tilde{\bar{S}}_d\right) = (0.1177, 0.1241, 0.1316, 0.1451)$$

$$\bar{\bar{X}} = \left(\bar{\bar{X}}_a, \bar{\bar{X}}_b, \bar{\bar{X}}_c, \bar{\bar{X}}_d\right) = (5.3711, 5.4167, 5.4612, 5.5096)$$

$$\bar{\bar{X}}_a^{0.65} = \bar{\bar{X}}_a + \alpha\left(\bar{\bar{X}}_b - \bar{\bar{X}}_a\right) = 5.4053$$

$$\bar{\bar{X}}_d^{0.65} = \bar{\bar{X}}_d - \alpha\left(\bar{\bar{X}}_d - \bar{\bar{X}}_c\right) = 5.4733$$

$$\tilde{\bar{S}}_a^{0.65} = \bar{S}_a + \alpha(\bar{S}_b - \bar{S}_a) = 0.1225$$

$$\tilde{\bar{S}}_d^{\alpha} = \bar{S}_d + \alpha\left(\tilde{\bar{S}}_d - \tilde{\bar{S}}_c\right) = 0.1350$$

The control limits and centre line using α - Level fuzzy midrange for α - Cut $\tilde{\bar{X}}$ control chart based on standard deviation are:

$$U\tilde{C}L_{mr-\bar{X}}^{0.65} = CL_{mr-\bar{X}}^{0.65} + A_3\left(\frac{\tilde{\bar{S}}_a^{0.65} + \tilde{\bar{S}}_d^{0.65}}{2}\right) = 5.6230$$

$$\tilde{C}L_{mr-\bar{X}}^{0.65} = f_{mr-\bar{X}}^{0.65}\left(\widetilde{CL}\right) = \frac{\bar{\bar{X}}_a^{0.65} + \bar{\bar{X}}_d^{0.65}}{2} = 5.4393$$

$$L\tilde{C}L_{mr-\bar{X}}^{0.65} = CL_{mr-\bar{X}}^{0.65} - A_3\left(\frac{\tilde{\bar{S}}_a^{0.65} + \tilde{\bar{S}}_d^{0.65}}{2}\right) = 5.2556$$

α - Level fuzzy midrange for α - Cut $\tilde{S}$ control chart

$U\tilde{C}L^{0.65}_{mr-\tilde{S}} = B_4 f^{0.65}_{mr-\tilde{S}}(\tilde{C}L) = 0.2689$

$\tilde{C}L^{0.65}_{mr-\tilde{S}} = f^{0.65}_{mr-\tilde{S}}(\tilde{C}L) = 0.1287$

$L\tilde{C}L^{0.65}_{mr-\tilde{S}} = B_3 f^{0.65}_{mr-\tilde{S}}(\tilde{C}L) = 0$

The values of $S^{\alpha}_{mr-\bar{X}.j}$ and $S^{\alpha}_{mr-S.j}$ have been calculated by using the formula α - Level fuzzy midrange for α-Cut $\tilde{\tilde{X}} - \tilde{S}$ control chart respectively and the values are given in Table 2.

Table 2. Control limits using α-level fuzzy midrange for α-cut fuzzy $\tilde{\tilde{X}}$ control chart based on standard deviation and α-level fuzzy midrange for α-cut fuzzy $\tilde{S}$ control chart

Sample No.	$S^{\alpha}_{mr-\bar{X}.j}$	$5.2556 < S^{\alpha}_{mr-\bar{X}.j} < 5.6230$	$S^{\alpha}_{mr-S.j}$	$0 < S^{\alpha}_{mr-S.j} < 0.2689$
1	5.5130	In Control	0.1756	In Control
2	5.5088	In Control	0.1532	In Control
3	5.3163	In Control	0.1262	In Control
4	5.5215	In Control	0.1305	In Control
5	5.3490	In Control	0.1259	In Control
6	5.4265	In Control	0.1116	In Control
7	5.3920	In Control	0.1140	In Control
8	5.5305	In Control	0.1157	In Control
9	5.4775	In Control	0.1736	In Control
10	5.3493	In Control	0.0745	In Control
11	5.4373	In Control	0.0934	In Control
12	5.3735	In Control	0.1420	In Control
13	5.4100	In Control	0.0918	In Control
14	5.4993	In Control	0.0844	In Control
15	5.4850	In Control	0.1536	In Control

Based on the results of table 2. above, it shows that nothing sample point that is out of control. Thus, we could say that the production process of piston is in control according to mean with α-level fuzzy midrange for α-cut fuzzy $\tilde{\tilde{X}}$ control chart based on standard deviation.

Summary

With this study it is shown that the fuzzy set theory is applicable on the traditional variable control charts and fuzzy control charts can provide more flexibility for controlling process.

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