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Exploring local plausible reasoning: the case of inequality tasks

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Exploring local plausible reasoning: the case of inequality tasks

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Abstract. This study explores local plausible reasoning in solving inequality tasks. The study was conducted to 78 students of mathematics education major from a university in Surabaya, Indonesia. Data were collected through written tasks and interviews. The data were analysed by a constant comparative method. The results of the study were the characteristics of local plausible reasoning shown by these behaviours: (1) students applied plausible reasoning in the local part of task solving or (2) students involved a conceptual understanding or a relational understanding in a few part of entirely argumentations. Educators can overcome the students' behaviours by designing a meaningful learning strategy which develops students' plausible reasoning in the whole of task solving.

1. Introduction

Existing studies about plausible reasoning (PR) have been widely documented in the literature [1-5]. Those studies relate to task solving. The result showed that a majority of students did not involve PR. In fact, PR is an essential competency to be applied in conjecturing, problem solving, and proving.

Polya [6] used the term PR to differentiate a more reasonable guess from a less reasonable guess. Meanwhile, Lithner [7] analysed the qualities of mathematical reasoning into 2 categories, namely PR and reasoning based on established experience (EE). Lithner [8, 9] added 2 new categories, namely local plausible reasoning (LPR) and global plausible reasoning (GPR). Thus, there are 4 types of mathematical reasoning, namely EE, LPR, GPR, and PR. The qualities of mathematical reasoning are not focused on a right or wrong answer, but they focus on the process of argumentations during task solving. Relating to Lithner's idea, the authors define EE as reasoning by giving argumentation based on ideas and procedures constructed from student's previous experience without deep understanding. This understanding refers to conceptual understanding [10] or relational understanding [11]. Meanwhile, PR is defined as reasoning by giving argumentation based on intrinsic mathematical properties. LPR is defined as PR applied locally in the whole of task solving, whereas GPR is defined as PR applied globally in the whole of task solving.

The intrinsic property is relevant to task solving or is accepted by mathematical society as a correct. It is vital in particular contexts and situations [8, 9]. The intrinsic property is opposite of a surface property. The surface property has no or little relevance to task solving. It is not the principal part of task solving because it does not consider the understanding of mathematical ideas. As an example, given that a fraction task to determine a greater fraction between $\frac{7}{16}$ and $\frac{3}{5}$. The quotient of the fraction is an intrinsic property, but the numerical values (7, 16, 3, and 5) are a surface property which is insufficient to be considered in the task solving. An argumentation is based on a surface property such as "$\frac{7}{16}$ is greater than $\frac{3}{5}$ because 7 and 16 are greater than 3 and 5", whereas an argumentation is based on an intrinsic property such as "since $\frac{7}{16}$ is equal to 0.4375 and $\frac{3}{5}$ is equal to 0.6, $\frac{3}{5}$ is greater than $\frac{7}{16}$." Other examples of argumentations based on intrinsic mathematical properties such as (1) "because in the fraction $\frac{7}{16}$ the value of the numerator is less than a half of the denominator, so the value of $\frac{7}{16}$ is less than $\frac{1}{2}$. Whereas in the fraction $\frac{3}{5}$ The value of the numerator is greater than a half of the denominator, so the value of $\frac{3}{5}$ is greater than $\frac{1}{2}$. Hence, $\frac{3}{5} > \frac{7}{16}$."

In the fraction task, student may get a true answer, but his/her argumentation based on established experience without deep understanding. For instance, "according to my teacher's explanation, the task can be solved by cross multiplication in two fractions $\frac{7}{16}$ and $\frac{3}{5}$ so that it is obtained $7 \times 5 = 35$ and $16 \times 3 = 48$. Because 35 is less than 48, so $\frac{7}{16}$ is less than $\frac{3}{5}$." Student only applies the rule and cannot give a reason conceptually. In this case, the student's reasoning is EE.

The results of several earlier studies on the qualities of mathematical reasoning indicated that many students used EE or LPR rather than GPR or PR [7-9]. Most students did not consider the intrinsic mathematical property in mathematical task solving. Those studies only identified the qualities of mathematical reasoning, but they did not examine the characteristics of LPR. Therefore, the aim of this study is to reveal LPR in the inequality task solving.

The inequality concept underlies the understanding of the concept in almost all mathematical fields [12]. Students who do not understand it clearly may lead to a great difficulty when dealing with concepts such as function, monotonicity and concavity. If students understand the inequality concept well, the tasks become easier to be solved. On the contrary, it is difficult for students to make sense the inequality concept.

The finding presented in this study will expand educators' knowledge about students' reasoning in the inequality task solving. This study is significant because it provides a contribution associated with the characteristics of LPR. The finding can be used by educators as a guide in designing the learning strategies or learning tools to improve students' LPR.

2. Literature review

Reasoning has become an essential topic to be promoted to students [13]. Students need to apply their reasoning in order to gain success in mathematics learning and mathematical task solving. By applying reasoning, mathematics can be meaningfully understood. Yet, the fact is that many students do rote learning. Students learn by memorising without knowing the meaning and the reason why a rule or a procedure works. Ultimately, their learning process becomes meaningless.

Reasoning can be seen as thought process, the product of thought process, or both [14]. In this study, the reasoning refers to the thinking process. The reasoning is the line of thinking used by someone to produce statements and reach a conclusion in task solving. To solve a mathematical task, a student can solve a set of subtasks which have the different character. The mathematical task can be categorised into two types, i.e. an exercise (a routine task) or a problem (a non-routine task). A mathematical task is classified as a problem if a student understands the mathematical task and desires to solve it (challenging to solve the task), but the complete solution scheme is not available immediately on the student's mind. While if the student has a complete procedure solution (a complete solution scheme) immediately, then the mathematical task is considered as a routine task. The routine task frequently can be solved by the direct application of one or more algorithms. The term algorithm means that a systematic procedure to solve a mathematical task.

Lithner [8, 9] defines PR in mathematical task solving if the argumentation: (i) is based on intrinsic mathematical properties of the component involved in the reasoning, and (ii) is intended for guiding toward what probably is the truth, without necessarily complete or correct. The component relates to mathematical objects (e.g. facts, concepts, definitions, operations, principles, or procedures) and heuristic. Reasoning in mathematical task solving is called as EE if the argumentation: (i) is based on the ideas and procedures established by the one's previous experience of the learning environment, and (ii) is intended for guiding toward what probably is the truth, without necessarily complete or correct [7-9]. Condition (ii) in the definition of EE is same as in the definition of PR because the goal of the reasoning is the same. The fundamental difference between the definition of PR and EE is on the argumentation as described by the condition (i). Generally the argumentation in EE is the transfer of property from one situation of familiar task solving to another situation that has some similarity. EE is superficial reasoning, while PR is mathematically well-founded reasoning [9].

PR and EE are the extension of analytical thought process and pseudo-analytical thought process introduced by Vinner [15]. The analytical thought process occurs when students faces a structure of a complicated problem and their scheme does not reach it, so they will solve the problem into simpler parts that can be reached out. The difference between the analytical thought process and PR is the degree of certainty in reasoning. The degree of certainty in PR is higher than the analytical thought process. Whereas, the difference between pseudo-analytical thought process and EE is on the degree of analyticity. The pseudo-analytical thought process is not the analytical thought process, but EE has little bit analytical thought content. Students who apply the pseudo-analytical thought process may produce a wrong or right solution.

Other types of mathematical reasoning are LPR and GPR. The difference between LPR and GPR is the range of PR, whereas the similarity is the existence of PR. GPR relates to applying PR globally from the entirely solution, while LPR relates to applying PR locally from the entirely solution. If a mathematical task is impossible to be solved by EE, LPR, or GPR, then PR in the entirely solution needs to be applied.

The authors further make the position of Lithner's reasoning characterisation based on the lens of PR in the whole of task solving. It includes EE, LPR, GPR, and PR. These four types of mathematical reasoning are a continuum. It can be illustrated in a straight line as presented in Figure 1. The two extremities of reasoning are EE and PR. EE is opposite of PR. PR represents the peak of reasoning with a rich network of argumentations based on intrinsic mathematical properties, whereas EE represents the lowest reasoning with argumentations based on the ideas and procedures from previous experience without a deep understanding.

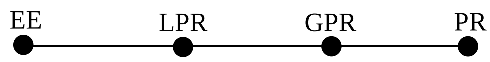


Figure 1. Continuum of mathematical reasoning

3. Method

This exploratory study with a qualitative approach was undertaken to second-year students of a private university located in Surabaya, Indonesia. The participants were 78 students, consisting 63 females and 15 males. The participants' age ranged from 18 to 20 years. All the participants were students of mathematics education major who attended multivariate calculus course. Data were obtained by giving an instrument of task sheet which consists 3 items of inequality tasks as shown in Figure 2. The participants solved the inequality tasks independently. Some notes were taken by the authors during the session of task solving.

Find the set of all real numbers x which satisfies the following inequality tasks.
Task 1. $3x^2 \leq 0$ Task 2. $x^2 < 2x$ Task 3. $2 - 8x < 4(1 - 2x)$

Figure 2. The instrument of task sheet

An interview was carried out for collecting data of students' argumentation. The authors recorded the students' conversation and behaviour during the interview. Further, the authors transcribed the recording of the interview. The authors analysed the data using a constant comparative method. It is called the constant comparative method because in this study the data analysis compares the data with the other data constantly, and then it compares the category with the other categories regularly [16, 17]. The method was done to get the characteristics of LPR.

4. Results and discussion

Distribution of students' reasoning in solving inequality tasks is shown in Table 1. In all tasks, many students applied LPR. The highest proportion of LPR was given in Task 3 (53% of the students), whereas the lowest proportion of LPR was given in Task 1 (38% of the students) and 44% of students applied LPR in Task 2. It is noteworthy that a few students applied PR in the whole solution of each task (17%, 10%, 6%).

Table 1. Distribution of students' reasoning in each task

Inequality task	Frequency (percentage) of students' reasoning				Total
	EE	LPR	GPR	PR	
Task 1	18 (23%)	30 (38%)	17 (22%)	13 (17%)	78 (100%)
Task 2	25 (32%)	34 (44%)	11 (14%)	8 (10%)	78 (100%)
Task 3	12 (15%)	41 (53%)	20 (26%)	5 (6%)	78 (100%)

The construction of LPR is next analysed based on students' responses in each task separately. Following the authors report and discuss the analysis of each task. In Task 1, many students considered intrinsic mathematical properties in the local part of task solving. For instance, S1 factorised $3x^2 \leq 0$ to $3x.x \leq 0$. Subsequently, S1 used her previous knowledge about the solution procedure of the equation to the solution procedure of inequality so that it is obtained $(3x \leq 0$ or $x \leq 0) \Leftrightarrow (x \leq 0$ or $x \leq 0)$. S1 determined the solution set of Task 1, i.e. $\{x|\ x \leq 0, x \in \mathbb{R}\}$. S1's written answer to Task 1 is shown in Figure 3.

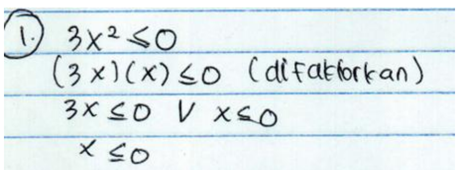


Figure 3. S1's written answer to Task 1

S1 applied PR locally on the part of the whole solution, i.e. involving the concept of factorisation. S1's argumentation related to applying the solution procedure based on her previous experience. If it is viewed from Vinner's terminology [15], S1 only identified the superficial similarity between the solution procedure of $3x.x = 0$ and the solution procedure of $3x.x \leq 0$. S1 did not understand that the solution procedure in the domain of the equation could not be applied in the domain of the inequality. This could result in the improper solution procedure. According to Tall's terminology [18-20], S1 did not realise that the met-before of the solution procedure in the domain of the equation does not work in the domain of the inequality. In other words, a problematic met-before occurred when S1 used met-before outside the domain of validity. Here is the excerpt of S1's argumentations in the interview process.

R: *Your solution set of Task 1 is* $\{x|\ x \leq 0, x \in \mathbb{R}\}$. *How can it be?*
S1: *I factorised* $3x^2 \leq 0$ *so that I got* $3x.x \leq 0$. *At this moment, I faced a problem because I did not know how to solve it. After I recalled my previous experience, I knew how to*

proceed. I considered the way of equation solving to be applied in the inequality solving. I got $3x \leq 0$ *or* $x \leq 0$*. For* $3x \leq 0$ *will be* $x \leq 0$*. Because the last part of the solution is* $x \leq 0$ *or* $x \leq 0$*, so the solution set is* $x \leq 0$*.*

R: *How did you guarantee that your way worked? Justify it.*

S1: *I often used this way to solve inequality. This way is same as a way to solve inequality.*

R: *How do you convince that your answer is correct?*

S1: *I was sure that my answer was correct because I used my previous knowledge about the way of equation solving.*

Note. R = Researcher

In Task 2, an example of student's LPR is represented by S2 behaviour. S2 added both sides of $x^2 < x$ by $-2x$ so that it is obtained $x^2 - 2x < 0$. S2 factorised $x^2 - 2x < 0$ to $x(x-2) < 0$. S2 solved $x(x-2) < 0$ by following the solution procedure from his teacher without relational understanding. S2 determined the values for which each factor is zero, i.e. $x = 0$ or $x = 2$. S2 made three intervals on a real number line, i.e. $(-\infty, 0)$, $(0,2)$, or $(2, \infty)$. S6 tested the values of x at each interval. S2 substituted each of the values $x = -1$, $x = 1$, and $x = 3$ to $f(x) = x(x-2)$ so that it is obtained $f(-1) = 3$ (positive), $f(1) = -1$ (negative), and $f(3) = 3$ (positive). Since $x(x-2) < 0$ has a negative value, S2 decided $\{x|\ 0 < x < 2, x \in \mathbb{R}\}$ as the solution set of Task 2. S2's written answer to Task 2 is shown in Figure 4.

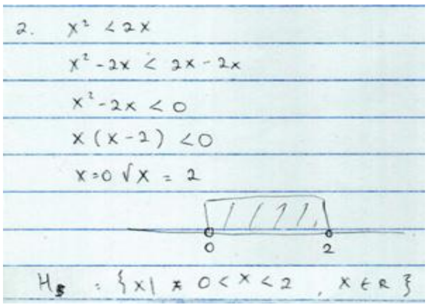


Figure 4. S2's written answer to Task 2

S2 applied PR locally on the part of the whole solution, i.e. involving the rule of adding both sides of the inequality by an expression and the concept of factorisation. S2 understood the rule and the concept. For the next solution steps, S2 applied the solution procedure based on his previous experience from his teacher or textbook. S2 only followed the solution procedure of inequality without knowing the reason why the procedure worked. In this task, S2 got the correct answer, but he was unable to show a right reasoning. According to Vinner's terminology [15], his thought process is categorised as pseudo-analytical. S2's argumentations which support the response are shown in the following excerpt.

R: *How did you get the solution set of Task 2? Explain it.*

S2: *I did it because I have memorised the solution procedure of the inequality. I have learned it from my teacher. I change the symbol "*$>$*" by "*$=$*" for finding the values of* x*. It can be determined the values for which each factor is zero.*

R: *Why can you change the symbol "*$>$*" by "*$=$*"? Justify it.*

S2: *I got the solution procedure of inequality from my teacher. My teacher said that to solve inequality task, we change the symbol of inequality by the symbol of the equation. I also studied the solution procedure of inequality in the textbook.*

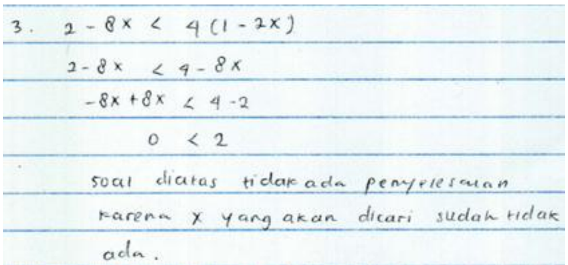
R: *Any other reasons? Convince me that your procedure works.*

S2: *I do not know, sir. I only apply the procedure from my learning experience.*

R: *How do you convince that all values in* $0 < x < 2$ *satisfy Task 2?*

S2: *I was very sure that my answer is correct because my teacher said that to find the solution set of the inequality we can do test value in the interval and take one value. If one value of x satisfies the inequality, we decide all values in the interval as the solution. Because for testing the value of x=1 in the $0 < x < 2$ satisfied the Task 2, so all values in $0 < x < 2$ also satisfy Task 2.*

In Task 3, an example of student's LPR is represented by S3 behaviour. S3 simplified $2 - 8x < 4(1 - 2x)$ to $2 - 8x < 4 - 8x$ by applying the distributive property of multiplication over addition. Afterwards, S3 grouped the expression on the left-hand side of the inequality, while the quantity was grouped on the right-hand side of the inequality. By applying the rule "moving side mean changing sign", S3 simplified the solution process so that it is obtained $0 < 2$ as the last part of the solution. This rule is often done by S3 on the solution of inequalities. This indicates that S3 only focuses on imitating the rule that has been taught by her teacher without the effort of relational understanding. S3 faced a problematic situation. S3 thought for a few minutes and solved task again. S3 was unable to continue the last step of the solution. S3 revealed that there is no solution for Task 3 because the variable x is missing. S3's written answer to Task 3 is shown in Figure 5.



Translation:
The task has no solution because there is no variable x.

Figure 5. S3's written answer to Task 3

S3 could not find the solution set of Task 3 because her conceptual understanding was poor. S3 did not conduct a controlled association in her reasoning process so that she failed to evaluate her reasoning. This reasoning association mechanism which involves the uncontrolled reaction and association without understanding is called as the uncontrolled association [15]. Actually, all real numbers x satisfies $2 - 8x < 4(1 - 2x)$. In other words, the solution set of Task 3 is a real number set. Here is the excerpt of S3's argumentations in the interview process.

R: *Could you explain your solution to Task 3?*
S3: *I distributed the right-hand side of $2 - 8x < 4(1 - 2x)$ so that it was obtained $2 - 8x < 4 - 8x$. I grouped the variable and the number to the left-hand side and right-hand side, respectively. I used the rule moving side mean changing the sign. I operated and got the result $0 < 2$.*
R: *How did the rule moving side mean changing the sign work? Give reason.*
S3: *I often solved inequalities tasks. The rule is always like that. I got it from my teacher in the past.*
R: *Could you convince that your answer is correct?*
S3: (The student is quiet for a few minutes and try to solve Task 3 again) *I was not sure, sir. I got stuck. I could not solve the task because there is no variable x in the last solution. I was confused to determine the solution set. The task was strange.*

The constant comparative method was used to analyse the data of students' LPR so that it was obtained the same characteristics of LPR. The characteristics were that (1) students applied plausible reasoning in the local part of task solving or (2) students' argumentations were mainly founded on their previous experience by considering a few conceptual understanding or relational understanding.

In general, the structure of LPR in task solving can be illustrated in Figure 6. The orange and purple circles represent argumentations, and the lines joining the argumentations represent the relationship or connections between argumentations. The purple circle is an argumentation based on intrinsic mathematical property, whereas the orange circle is an argumentation based on ideas and procedures from previous experience without deep understanding.

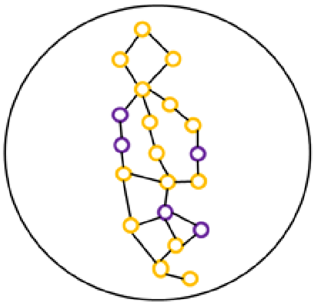


Figure 6. Structure of local plausible reasoning in task solving

These results extend previous results [1-9] to a new situation, namely exploring the characteristics of local plausible reasoning rather than investigating students' difficulty or misconception. Besides, the present study showed that the highest percentage of students' local plausible reasoning was given in the inequality task which has a real number set as the solution set. Such task was called by the students as a strange task or an undefined task. Students experienced an extreme difficulty. Some students stated that (1) the value of x cannot be determined, (2) the task cannot be solved, or (3) the task has no solution set. The finding is accordance with the results of existing studies [2, 3] which reported that an inequality producing a real number set for the solution is very problematic for students. Another finding of the present study is that some students had an intuitive belief that equations and inequalities can be solved by the same procedures. The finding is consistent with earlier studies [4, 5] which reported that the effect of the intuitive belief solving inequalities and equations are the same was extensively evident in many students' solution.

For educational implications, a meaningful learning strategy seems to be an effective way to teach and learn the inequality concept. By the meaningful learning strategy, students can understand the new knowledge well by relating to their existing knowledge. Students' reasoning process is developed through the process of meaningful learning [21]. In addition, a process-oriented instruction can be implemented in mathematics classrooms to promote students' mental models. It is an instructional model in which students are taught thinking strategies to construct, modify and utilise their knowledge or mental models [22].

In this study, some students used their gesture to clarify their reasoning. For further research, it is crucial to examine the contribution of gestures toward students' reasoning. This is supported by the fact that gestures can be utilised as a tool for reasoning [23-25]. The influence of gender is not investigated in this study. Therefore, comparisons of students' local plausible reasoning ability based on gender differences are possible in a future study.

5. Conclusion

In summary, the present study showed that the proportion of local plausible reasoning was dominant. Most students focused on mimicking the rules or procedures without understanding the inequality per se. Moreover, students did not realise the mistake in applying the solution procedure of the equation to the solution procedure of inequality. Therefore, in the instructional process educators should train and develop students' plausible reasoning optimally. Students need to be familiarised for explaining the process of task solving, justifying the steps of task solving (i.e. giving reasons why the students think or answer like that, why students use this strategy, or why the rule/procedure works), and convincing the truth of the obtained results. In addition, educators also can provide scaffolding for students who apply local plausible reasoning in order to have plausible reasoning in the whole of solution task.

References

[1] Almog N and Ilany B S 2012 *Educational Studies in Mathematics* **81** 347
[2] Fujii T 2003 *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education* vol 1 eds N. A. Pateman et al (Honolulu, HI: University of Hawai‘i) pp 49–65
[3] Tsamir P and Almog N 1999 *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education* (Haifa, Israel: PME) p 328
[4] Tsamir P and Almog N 2001 *International Journal of Mathematical Education in Science and Technology* **32** 513
[5] Tsamir P and Bazzini L 2004 *International Journal of Mathematical Education in Science and Technology* **35** 793
[6] Polya G 1954 *Mathematics and Plausible Reasoning: Vol. I. Induction and Analogy in Mathematics* (Princeton, New Jersey: Princeton University Press)
[7] Lithner J 2000 *Educational Studies in Mathematics* **41** 165
[8] Lithner J 2003 *Educational Studies in Mathematics* **52** 29
[9] Lithner J 2004 *Journal of Mathematical Behavior* **23** 405
[10] Hiebert J and Carpenter T P 1992 *Handbook of Research on Mathematics Teaching and Learning* ed D. A. Grouws (New York: Macmillan Publishing Company) pp 65–97
[11] Skemp R R 1976 *Mathematics Teaching* **77** 20
[12] Rofiki I, Nusantara T, Subanji and Chandra T D 2017 *IOSR Journal of Research & Method in Education* **7** 101
[13] Rofiki I, Subanji, Nusantara T and Chandra T D 2015 *Proceeding of International Conference on Educational Research and Development* (Surabaya: State University of Surabaya) pp 358-365
[14] Lithner J 2008 *Educational Studies in Mathematics* **67** 255
[15] Vinner S 1997 *Educational Studies in Mathematics* **34** 97
[16] Creswell J W 2012 *Educational Research: Planning, Conducting, and Evaluating Quantitative and Qualitative Research* (Boston: Pearson Education, Inc)
[17] Kolb M S 2012 *Journal of Emerging Trends in Educational Research and Policy Studies (JETERAPS)* **3** 83
[18] Tall D O 2008 *Mathematics Education Research Journal* **20** 5
[19] Lima R N de and Tall D O 2008 *Educational Studies in Mathematics* **67** 3
[20] Tall D O, Lima R N de and Healy L 2014 *Journal of Mathematical Behavior* **34** 1
[21] Ausubel D P 2000 The Acquisition and Retention of Knowledge: A Cognitive View (Dordrecht: Springer-Science+Business Media, B.V.)
[22] Vermunt J D 1995 *European Journal of Psychology of Education* **10** 325
[23] Soto-Johnson H and Troup J 2014 *Journal of Mathematical Behavior* **36** 109
[24] Williams-Pierce C, Pier E L, Walkington C, Boncoddo R, Clinton V, Alibali M W and Nathan M J 2017 *Journal for Research in Mathematics Education* **48** 248
[25] Nurhasanah F, Kusumah Y S and Sabandar J 2017 Concept of triangle: Examples of mathematical abstraction in two different contexts *International Journal on Emerging Mathematics Education* **1** 53