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On the distance spectrum and distance energy of complement of subgroup graphs of dihedral group

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Abstract. Let G is a connected simple graph and $V(G) = \{v_1, v_2, \dots, v_p\}$ is vertex set of G . The distance matrix of G is a matrix $D(G) = [d_{ij}]$ of order p where $[d_{ij}] = d(v_i, v_j)$ is distance between v_i and v_j in G . The set of all eigenvalues of matrix $D(G)$ together with their corresponding multiplicities is named the distance spectrum of G and denoted by $spec_D(G)$. The distance energy of G is $E_D(G) = \sum_{i=1}^p |\lambda_i|$, where λ_i are eigenvalues of $D(G)$. In the recent paper, the distance spectrum and distance energy of complement of subgroup graphs of dihedral group are determined.

1. Introduction

The study on spectrum of a graph begins with the work of Bigg [1] and defined on the adjacency matrix of graph. Next developed concept of spectrum based on various matrices obtained from a graph, such as detour spectrum [2], Laplacian spectrum [3,4], signless Laplacian spectrum [5,6], distance spectrum [7], detour distance Laplacian spectrum [8] and color signless Laplacian spectrum [9]. Until now the topic of distance spectrum is still extensively studied by researchers, see [10-21].

The concept that is very close to the concept of spectrum of a graph is energy. The concept of energy of graph was first proposed by Gutman [22]. Adjacency energy is the first concept of energy that is discussed and has many benefits in the field of chemistry [12]. Further developed other concepts of energy that widely used in graphs, such as incidence energy [23], Laplacian incidence energy [24], maximum degree energy [25], matching energy [26], Harary energy [27], color energy [28] and distance energy [29]. Distance energy was first proposed by Indulal and Gutman [7,29]. Research on the distance energy of a connected simple graph also got a great attention from the researchers, see [13,30-38].

The most recent development in graph theory is the topic of graph associated with a group. Several concepts of graph obtained from a group have been introduced, for example Cayley graph [39], identity graph [40], commuting graph [41], conjugate graph [42], non commuting graph [43], subgroup graph [44] and inverse graph [45] of a group. For given normal subgroup H of a finite group G , the subgroup graph $\Gamma_H(G)$ of G is defined as simple graph with $V(\Gamma_H(G)) = G$ and $xy \in E(\Gamma_H(G))$ if and only if $xy \in H$ [43,45].



From its introduction in 2012 by Anderson et al [44], research on subgroup graph is still rare and some of them are [45,46]. So, research on the distance spectrum and energy of subgroup graph is an interesting topic to be conducted.

2. Literature Review

Graph in the present paper is simple and finite. Graph G is connected if there exist u - v path in G , for any two vertices u and v in G [48]. Graph \bar{G} with $V(\bar{G}) = V(G)$ and $xy \in E(\bar{G})$ if and only if $xy \notin E(G)$ is named as complement of G [49]. Graph G is complete if for any two distinct vertices $x, y \in V(G)$ then $xy \in E(G)$. If $V(G)$ can be partitioned into $k > 1$ partition sets such that an edge of G joining two vertices in the different partition then G is said to be k -partite graph. A k -partite graph G is called to be complete if for every vertices in different partition set are adjacent [50]. If $V_1, V_2, V_3, \dots, V_k$ are partition set of a complete k -partite graph and $|V_i| = n_i$ then this graph is denoted by $K_{n_1, n_2, n_3, \dots, n_k}$.

Let graph G is connected with $V(G) = \{v_1, v_2, \dots, v_p\}$ as its vertex set. The distance matrix of G is a $p \times p$ matrix $D(G) = [d_{ij}]$ such that $[d_{ij}] = d(v_i, v_j)$, where $d(v_i, v_j)$ is the length of the shortest v_i - v_j path in graph G . The characteristics polynomial of $D(G)$ is $p_D(\lambda) = \det(D(G) - \lambda I)$ and the roots of the characteristics equation are named eigenvalues of $D(G)$ or simply D -eigenvalues of graph G . The set of all D -eigenvalues of G together with their multiplicities is named the distance spectrum or simply D -spectrum of G and notated by $spec_D(G)$ [11]. Since D -matrix is real and symmetric then all D -eigenvalues are also real [34]. Let $\lambda_1 > \lambda_2 > \dots > \lambda_k$ are distinct D -eigenvalues of G with multiplicities m_1, m_2, \dots, m_k . According to the ordinary spectrum notation [51], then D -spectrum of G is notated by

$$spec_D(G) = \begin{bmatrix} \lambda_1 & \lambda_2 & \dots & \lambda_k \\ m_1 & m_2 & \dots & m_k \end{bmatrix}.$$

The distance energy or simply D -energy $E_D(G)$ of connected graph G is defined as

$$E_D(G) = \sum_{i=1}^p |\lambda_i| = \sum_{j=1}^k m_j |\lambda_j|.$$

Some previous results on D -spectrum and D -energy of a connected graphs that will useful for further discussion as the followings.

Theorem 1.1. ([51], Theorem 4.1.)

Let $K_{n_1, n_2, n_3, \dots, n_k}$ is complete k -partite graph of order $n = \sum_{i=1}^k n_i$. Then, characteristic polynomial of $D(K_{n_1, n_2, n_3, \dots, n_k})$ is

$$p_D(\lambda) = (\lambda + 2)^{n-k} \left[\prod_{i=1}^k (\lambda - n_i + 2) - \sum_{i=1}^k n_i \prod_{j=1, j \neq i}^k (\lambda - n_j + 2) \right].$$

The next result was first conjectured by [52] and then proved by [53].

Theorem 1.2.

If $n_1, n_2, \dots, n_k > 2$ then $E_D(K_{n_1, n_2, n_3, \dots, n_k}) = 4(n_1 + n_2 + \dots + n_k - k)$.

Based on Theorem 1.1. the following corollaries are obvious.

Corollary 1.3. ([12], Corollary 3.)

Distance spectrum of complete bipartite graph K_{n_1, n_2} is

$$spec_D(K_{n_1, n_2}) = \begin{bmatrix} n_1 + n_2 - 2 + \sqrt{n_1^2 - n_1 n_2 - n_2^2} & n_1 + n_2 - 2 + \sqrt{n_1^2 - n_1 n_2 - n_2^2} & -2 \\ 1 & 1 & n_1 + n_2 - 2 \end{bmatrix}$$

Corollary 1.4. ([12], Corollary 4.)

For $n_1, n_2 > 2$, $E_D(K_{n_1, n_2}) = 4(n_1 + n_2 - 2)$.

3. Main Results

Here, we focused on the study of subgroup graphs of dihedral group D_{2m} of order $2m$. It was well known that for odd n , the normal subgroup of D_{2m} are $\langle 1 \rangle$, $\langle r^d \rangle$ where $d|m$ and D_{2n} itself and for even m , the normal subgroups of D_{2m} are $\langle 1 \rangle$, $\langle r^d \rangle$ where $d|m$, $\langle r^2, s \rangle$, $\langle r^2, rs \rangle$ and D_{2m} itself. Because the concept of D -spectrum and D -energy of graph are only for connected graphs, we focused on the complement of these subgroup graphs when these subgroup graphs are not connected. The results of this study as follows.

Theorem 3.1.

If $m > 3$ then

$$(a) \text{spec}_D(\overline{\Gamma_{\langle r \rangle}(D_{2m})}) = \begin{bmatrix} 3m-2 & m-2 & -2 \\ 1 & 1 & 2(m-1) \end{bmatrix}.$$

$$(b) E_D(\overline{\Gamma_{\langle r \rangle}(D_{2m})}) = 8(m-1).$$

Proof.

(a) Since $\Gamma_{\langle r \rangle}(D_{2m}) = 2K_m$, we have $\overline{\Gamma_{\langle r \rangle}(D_{2m})} = K_{m,m}$. By Corollary 1.3. it is obvious that

$$\text{spec}_D(\overline{\Gamma_{\langle r \rangle}(D_{2m})}) = \begin{bmatrix} 3m-2 & m-2 & -2 \\ 1 & 1 & 2(m-1) \end{bmatrix}.$$

(b) Based on the proof of (a) and because $m > 3$, by Corollary 1.4. it is complete the proof.

Theorem 3.2.

If $m > 3$ and m is even then

$$(a) \text{spec}_D(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}) = \begin{bmatrix} \frac{5m-4}{2} & \frac{m-4}{2} & -2 \\ 1 & 3 & 2(m-2) \end{bmatrix}.$$

$$(b) E_D(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}) = 8(m-2).$$

Proof.

(a) For even $m > 3$, $\Gamma_{\langle r^2 \rangle}(D_{2m}) = 4K_{m/2}$. So, we obtain its complement is $\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})} = K_{m/2, m/2, m/2, m/2}$. Applying Theorem 1.1. and performing some computation we have

$$p_D(\lambda) = (\lambda + 2)^{2(m-2)} \left(\lambda - \frac{5m-4}{2} \right) \left(\lambda - \frac{m-4}{2} \right)^3.$$

The D -eigenvalues of $\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}$ are $(5m-4)/2$, $(m-4)/2$ and -2 and their multiplicities are 1, 3 and $2(m-2)$. Then,

$$\text{spec}_D(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}) = \begin{bmatrix} \frac{5m-4}{2} & \frac{m-4}{2} & -2 \\ 1 & 3 & 2m-4 \end{bmatrix}.$$

(b) From (a), then $E_D(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}) = \left| \frac{5m-4}{2} \right| + 3 \left| \frac{m-4}{2} \right| + 2(m-2)|-2| = 8(m-2)$. We can also apply Theorem 1.2. when $m > 4$ to get $E_D(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}) = 4 \left(4 \frac{m}{2} - 4 \right) = 8(m-2)$.

Theorem 3.3.

If $m > 3$ and m is even then

$$(a) \text{spec}_D(\overline{\Gamma_{\langle r^2, s \rangle}(D_{2m})}) = \begin{bmatrix} 3m-2 & m-2 & -2 \\ 1 & 1 & 2(m-1) \end{bmatrix}.$$

$$(b) E_D(\overline{\Gamma_{\langle r^2, s \rangle}(D_{2m})}) = 8(m-1).$$

Proof.

(a) Since $\Gamma_{\langle r^2, s \rangle}(D_{2m}) = K_m \cup K_m$, we have $\overline{\Gamma_{\langle r^2, s \rangle}(D_{2m})} = K_{m,m}$. By Corollary 1.3. the proof is clear.

(b) Based on the proof of (a) and because $m > 3$, by Corollary 1.4. the proof is obvious.

Theorem 3.4.

If $m > 3$ and m is even then

$$(a) \operatorname{spec}_D(\overline{\Gamma_{\langle r^2, rs \rangle}(D_{2m})}) = \begin{bmatrix} 3m-2 & m-2 & -2 \\ 1 & 1 & 2(m-1) \end{bmatrix}.$$

$$(b) E_D(\overline{\Gamma_{\langle r^2, rs \rangle}(D_{2m})}) = 8(m-1).$$

Proof.

(a) Since $\Gamma_{\langle r^2, rs \rangle}(D_{2m}) = K_m \cup K_m$, we have $\overline{\Gamma_{\langle r^2, rs \rangle}(D_{2m})} = K_{m,m}$. By Corollary 1.3. we prove the part (a).

(b) Based on the proof of (a) and because $m > 3$, by Corollary 1.4. the proof is clear.

4. Conclusions

We have computed distance spectrum and energy of complement of several subgroup graphs of dihedral group D_{2m} . The remaining subgroup graphs of dihedral group D_{2m} are given to the reader for further investigation.

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