Topological Indices of a Graph

Lecture Note

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Let $G$ be a graph with vertex set $V(G)$ and edge set $E(G)$.

The degree $\deg(v)$ of a vertex $v$ in $G$ is the number of vertices adjacent to $v$.

The set of all vertices which adjacent to $v$ is called open neighborhood of $v$ and denoted by $N(v)$.

We obtain that

$$\deg(v) = |N(v)|$$
The closed neighborhood of $v$ is the set
$$N[v] = N(v) \cup \{v\}.$$ 

The ve-degree $\text{deg}_{ve}(v)$ of a vertex $v$ in a graph $G$ is the number of different edges that incident to any vertex from the closed neighborhood of $v$. 

**ve-Degree of Vertex in Graph**
First Zagreb Index

\[ M_1(G) = \sum_{u \in V(G)} \deg(u)^2 = \sum_{uv \in E(G)} [\deg(u) + \deg(v)] \]

Second Zagreb Index

\[ M_2(G) = \sum_{uv \in E(G)} \deg(u) \deg(v) \]
Degree Based Topological Indices

First General Zagreb Index

\[ M_1^\alpha (G) = \sum_{u \in V(G)} \deg(u)^\alpha = \sum_{uv \in E(G)} [\deg(u)^{\alpha-1} + \deg(v)^{\alpha-1}] \]
First Zagreb Coindex

\[ M_1(G) = \sum_{uv \notin E(G)} [\deg(u) + \deg(v)] \]

Second Zagreb Coindex

\[ M_2(G) = \sum_{uv \notin E(G)} \deg(u) \deg(v) \]
Forgotten Topological Index or F-index

\[ F(G) = \sum_{u \in V(G)} \text{deg}(u)^3 = \sum_{uv \in E(G)} [\text{deg}(u)^2 + \text{deg}(v)^2] \]
First ve-degree Zagreb Index

\[ M_{ve_1}(G) = \sum_{uv \in E(G)} [\text{deg}_{ve}(u) + \text{deg}_{ve}(v)] \]

Second ve-degree Zagreb Index

\[ M_{ve_2}(G) = \sum_{uv \in E(G)} \text{deg}_{ve}(u) \text{deg}_{ve}(v) \]
F-ve-degree Index

\[ F_{ve}(G) = \sum_{u \in V(G)} deg_{ve}(u)^3 = \sum_{uv \in E(G)} [deg_{ve}(u)^2 + deg_{ve}(v)^2] \]
Reduced Second Zagreb index

\[ RM_2(G) = \sum_{uv \in E(G)} [\deg(u) - 1][\deg(v) - 1] \]

It implies that \( RM_2(G) = M_2(G) - M_1(G) + m \)
Narumi-Katayama Index

\[ NK(G) = \prod_{v \in V(G)} \text{deg}(v) \]

First Multiplicative Zagreb Index

\[ \Pi_1(G) = \prod_{v \in V(G)} \text{deg}(v)^2 = \prod_{uv \in E(G)} (\text{deg}(u) + \text{deg}(v)) \]

and we have \( \Pi_1(G) = NK(G)^2 \)

Second Multiplicative Zagreb Index

\[ \Pi_2(G) = \prod_{uv \in E(G)} \text{deg}(u) \text{deg}(v) \]
Randic Index or Connectivity Index or Branching Index or Product-Connectivity Index (1975)

\[ R(G) = \sum_{uv \in E(G)} \frac{1}{\text{deg}(u)\text{deg}(v)} = \sum_{uv \in E(G)} (\text{deg}(u)\text{deg}(v))^{-1} \]

and

\[ R(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\text{deg}(u)\text{deg}(v)}} = \sum_{uv \in E(G)} (\text{deg}(u)\text{deg}(v))^{-\frac{1}{2}} \]
Degree Based Topological Indices

Reciprocal Randic Index

\[ R(G) = \sum_{uv \in E(G)} \sqrt{\text{deg}(u)\text{deg}(v)} \]

Reduced Reciprocal Randic Index

\[ RRR(G) = \sum_{uv \in E(G)} \sqrt{[\text{deg}(u) - 1][\text{deg}(v) - 1]} \]
The General Randic Index

\[ R_\alpha(G) = \sum_{uv \in E(G)} (\deg(u) \deg(v))^\alpha \]

for an arbitrary \( \alpha \in \mathbb{R} \).

When \( \alpha = -1 \) or \( \alpha = -\frac{1}{2} \), we have the original Randic Index. When \( \alpha = 1 \), we have the Second Zagreb Index.
The General Product-Connectivity Coindex

\[ \overline{R}_\alpha(G) = \sum_{uv \notin E(G)} (\deg(u) \deg(v))^\alpha \]

for an arbitrary \( \alpha \in \mathbb{R} \).
Atom-Bond-Connectivity Index

$$ABC(G) = \sum_{uv\in E(G)} \sqrt{\frac{\text{deg}(u) + \text{deg}(v) - 2}{\text{deg}(u)\text{deg}(v)}}$$

Harmonic Index

$$H(G) = \sum_{uv\in E(G)} \frac{2}{\text{deg}(u) + \text{deg}(v)}$$
Augmented Zagreb Index

$$AZI(G) = \sum_{uv \in E(G)} \left( \frac{\text{deg}(u) \text{deg}(v)}{\text{deg}(u) + \text{deg}(v) - 2} \right)^3$$

Geometric-Arithmetic Index or First Geometric-Arithmetic Index

$$GA(G) = \sum_{uv \in E(G)} \frac{\sqrt{\text{deg}(u) \text{deg}(v)}}{\text{deg}(u) + \text{deg}(v)} = \sum_{uv \in E(G)} \frac{2\sqrt{\text{deg}(u) \text{deg}(v)}}{\text{deg}(u) + \text{deg}(v)}$$
Degree Based Topological Indices

Sum-Connectivity Index

\[ SCI(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\text{deg}(u) + \text{deg}(v)}} = \sum_{uv \in E(G)} (\text{deg}(u) + \text{deg}(v))^{-\frac{1}{2}} \]
General Sum-Connectivity Index

$$SCI_\alpha(G) = \sum_{uv \in E(G)} (\deg(u) + \deg(v))^\alpha$$

where $\alpha$ is any real numbers.

General Sum-Connectivity Coindex

$$\overline{SCI}_\alpha(G) = \sum_{uv \notin E(G)} (\deg(u) + \deg(v))^\alpha$$

where $\alpha$ is any real numbers.
For all unordered pairs of vertices in graph $G$.

Wiener Index

$$W(G) = \sum_{\{u,v\} \subseteq V(G)} d(u,v)$$

Wiener Polarity Index

$$W_P(G) = d(G, 3)$$

The number of vertex pairs of $G$ whose the distance is $k$ is denoted by $d(G, k)$. 
Terminal Wiener Index

\[ TW(G) = \sum_{\{u,v\} \subseteq V_1(G)} d(u, v) \]

where \( V_1(G) \subseteq V(G) \) is the set of vertices of \( G \) whose degree is equal to one (the so-called pendent vertices or leaves).
Hyper Wiener Index

\[ WW(G) = \frac{1}{2} \sum_{\{u,v\} \subseteq V(G)} [d(u, v) + d(u, v)^2] \]

Reciprocal Complementary Wiener Index

\[ RCW(G) = \sum_{\{u,v\} \subseteq V(G)} \frac{1}{diam(G) - 1 + d(u, v)} \]

Diameter of \( G \) is denoted by \( diam(G) \) and is defined as the greatest distance between any two vertices in \( G \).
Old Harary Index

\[ H_{old}(G) = \sum_{\{u,v\} \subseteq V(G)} \frac{1}{d(u, v)^2} \]

Harary Index

\[ H(G) = \sum_{\{u,v\} \subseteq V(G)} \frac{1}{d(u, v)} \]
Eccentricity Based Topological Indices

The eccentricity $e(v)$ of a vertex $v$ is the maximum distance from $v$ to any other vertex in $G$.

Total Eccentricity

$$
\xi(G) = \sum_{v \in V(G)} e(v)
$$
First Zagreb Eccentricity Index

$$E_1(G) = \sum_{u \in V(G)} (e(u))^2$$

Second Zagreb Eccentricity Index

$$E_2(G) = \sum_{uv \in E(G)} e(u) e(v)$$
Eccentric Connectivity Index

\[ \xi^c(G) = \sum_{v \in V(G)} e(v) \text{deg}(v) \]

Connective Eccentricity Index

\[ C^\xi(G) = \sum_{v \in V(G)} \frac{\text{deg}(v)}{e(v)} \]
$D(v)$ is defined as the sum of all distances from $v$ in $G$.

Eccentric Distance Sum Index

$$\xi^d(G) = \sum_{v \in V(G)} e(v)D(v)$$

or

$$\xi^d(G) = \sum_{\{u, v\} \subseteq V(G)} [e(u) + e(v)]d(u, v)$$
Adjacent Eccentric Distance Sum Index

\[ \xi^{sv}(G) = \sum_{v \in V(G)} \frac{e(v)D(v)}{\deg(v)} \]
Degree Distance Index or Schultz Index

\[ DD(G) = \sum_{u \neq v} (\text{deg}(u) + \text{deg}(v))d(u, v) \]

Gutman Index

\[ \text{Gut}(G) = \sum_{u \neq v} \text{deg}(u) \text{deg}(v)d(u, v) \]
Distance Degree Based Topological Indices

Additively Weighted Harary Index or Reciprocal Degree Distance Index

\[ H_A(G) = \sum_{u \neq v} \frac{(\text{deg}(u) + \text{deg}(v))}{d(u, v)} \]

Multiplicatively Weighted Harary Index

\[ H_M(G) = \sum_{u \neq v} \frac{\text{deg}(u) \text{deg}(v)}{d(u, v)} \]
REFERENCES


