GRAPHS ASSOCIATED WITH A COMMUTATIVE RING

LECTURE NOTE

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R will always denote a commutative ring.

An element $z \in R$ is called a zero divisor if there exists a non-zero $r \in R$ such that $rz = 0$.

An element $z \in R$ is called a unit if there exists $r \in R$ such that $rz = 1$.

The set of zero divisors is denoted $Z(R)$

The set of nonzero zero divisors as $Z(R)\setminus\{0\}$ or $Z(R)^*$.

The set of unit elements of R is denoted $U(R)$.

The annihilator of a ring element r is the set of all $s \in R$ such that $rs = 0$.

An element is said to be regular if it is not a zero divisor.

An element r is nilpotent if $r^n = 0$ for some positive integer n.

The Zero Divisor Graph

Let R be a commutative ring with identity which is not an integral domain.
The zero divisor graph of R, denoted by $\Gamma(R)$, is the graph whose vertex set is the set of all non-zero zero-divisors of R and distinct vertices x, y are joined by an edge in this graph if and only if $xy = 0$.

International Electronic Journal of Algebra
Volume 23 (2018) 176-202

THE ZERO-DIVISOR GRAPH OF A COMMUTATIVE RING WITHOUT IDENTITY
David F. Anderson and Darrin Weber

The Total Graph

The total graph of a commutative ring R as the graph with vertices of all elements of R and two distinct vertices $x, y \in R$ are adjacent if and only if $x + y$ is a zero-divisor in R or $x + y \in Z(R)$.

J. Algebra, 320(7):2706–2719, 2008.

The total graph of a commutative ring
David F. Anderson and Ayman Badawi

The Total Zero Divisor Graph

Denote the set of all zero divisors of R by Z(R).

The total zero divisor graph ZT(R) of a commutative unital ring R is a graph whose vertex set is the set of all non-zero zero divisors of R, and where distinct vertices u and v are adjacent if and only if

$$uv = 0 \text{ and } u + v \in Z(R).$$

Journal of Algebra and Its Applications · March 2018

The Total Zero-divisor Graph of Commutative Rings
Alen Duric, Sara Jevdenic, Polona Oblak, And Nik Stopar

The Co-zero Divisor Graph

Let R be a commutative ring with unity.
The co-zero divisor graph of R denoted by $\Gamma'(R)$ is a graph with the vertex set $W^*(R)$, where $W^*(R)$ is the set of all non-zero and non-unit elements of R, and two distinct vertices a and b are adjacent if and only if $a \notin Rb$ and $b \notin Ra$.

Journal of Algebra and Its Applications VOL. 13, NO. 03 2014

SOME RESULTS ON COZERO-DIVISOR GRAPH OF A COMMUTATIVE RING

S. AKBARI, F. ALIZADEH and S. KHOJASTEH

The Unit Graph

The unit graph of R is the simple graph with all elements of R as vertices, and two distinct vertices x and y are adjacent if and only if $x + y \in U(R)$.

August 2011. Algebra Colloquium 22 (spec01)

ON THE UNIT GRAPH OF A NONCOMMUTATIVE RING

S. AKBARI, E. ESTAJI, AND M.R. KHORSANDI

The Identity Graph

The identity graph of a commutative R is graph with all elements of U(R) as vertices, and two distinct vertices x and y are adjacent if and only if xy = 1.

Kandasamy, W. B. Vasantha and Smarandache, Florentin
Groups as Graphs
2009

Generalized Total Graph

Let S be a multiplicatively closed subset of ring commutative R.
Define the vertices of the graph to be the elements of R and (a, b) ∈ E if and only if a + b ∈ S.

If S is the set of zero divisors then this is the Total Graph.

If S is the set of units of R, this is the Unit Graph.

The Nilradical and Non-nilradical graph

The nilradical graph, denoted N(R), is the graph whose vertices are the non-zero nilpotents of R and where two vertices are connected by an edge if and only if their product is 0.

The non-nilradical graph, denoted Ω(R), is the graph whose vertices are the non-nilpotent zero-divisors of R and where two vertices are connected by an edge if and only if their product is 0.

International Journal of Algebra, Vol. 2, 2008, no. 20, 981 - 994

The Nilradical and Non-Nilradical Graphs of Commutative Rings
Abigail Bishop, Tom Cuchta, Kathryn Lokken, and Oliver Pechenik

The Annihilator Graph

The annihilator graph AG(R) of a commutative ring R is a simple undirected graph with the vertex set Z(R)* and two distinct vertices are adjacent if and only if $ann(x) \cup ann(y) \neq ann(xy)$.

Transactions on Combinatorics
Vol. 6 no. 1 (2017), pp. 1-11.

On annihilator graphs of a finite commutative ring
Sanghita Dutta and Chanlemki Lanong

The Maximal Graph

For any ring R, we associate a simple graph with vertices as the elements of R such that two different vertices x and y are adjacent if and only if $x, y \in M$, for some maximal ideal M of R. We call this graph as a maximal graph associated with R.

It is easy to see that every maximal ideal in R forms a maximally complete subgraph of maximal graph associated with R.

International Journal of Algebra, Vol. 7, 2013, no. 12, 581 – 588

Maximal Graph of a Commutative Ring
Atul Gaur and Arti Sharma

The Co-maximal Graph

Let R be a commutative ring with identity element.
Co-maximal graph on R, denoted by C(R), with all elements of R being the vertices of C(R), where two distinct vertices a and b are adjacent if and only if $aR + bR = R$

Linear Algebra and its Applications 437 (2012) 1040–1049.

Generalized Cayley graphs associated to commutative rings
Mojgan Afkhami, Kazem Khashyarmanesh, and Khosro Nafar

The Containment Graph of Ideals

Let R be a commutative ring with unity.
We define the containment graph Ω of R if the vertex set is the set of the ideals of R and two vertices I and J have an edge between them if and only if $I \subset J$

Journal of Mathematics and Statistics 2016, 12 (4): 308-311

On Containment and Radical Graphs of a Commutative Ring
Saba Al-Kaseasbeh

The Intersection Graph of Ideals

The intersection graph of ideals of a ring is a simple graph whose vertices are the nontrivial proper ideals and two vertices I and J are adjacent if $I \neq J$ and $I \cap J \neq \{0\}$.

International Journal of Combinatorics
Volume 2014, Article ID 952371, 6 pages

Some Properties of the Intersection Graph for Finite Commutative Principal Ideal Rings
Emad Abu Osba, Salah Al-Addasi, and Omar Abughneim