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## Investigating students' learning trajectory: a case on triangle

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**Abstract.** Learning trajectory becomes the main issues in mathematics education research. However, there have been limited studies of students' learning trajectory for triangle construction. Therefore, this design research was conducted to investigate students' learning trajectory for the topic of the triangle. The study involved 22 students from 7<sup>th</sup> grade in Malang, Indonesia. Data were collected through a videotaped, a student's worksheet, and a classroom observation. The results showed that students discovered the requirement of forming a triangle given three side lengths. In this condition, the starting point of students' learning trajectory was the drawing of a line segment from the given three side lengths. Students examined two side lengths whether these side lengths can be joined to the line segment as a triangle or not. Students used rulers for determining those three side lengths that could form a triangle. They made a statement that the sum of any two sides of a triangle must be greater than the third side. Furthermore, teachers should consider students' learning trajectory for achieving successfully the learning goal.

### 1. Introduction

Learning trajectory has been studied by researchers. Many studies on this issue have been conducted for several mathematics topics such as measurement [1,2], statistics [3], geometry [4,5], logic [6], functional relationships [7], fraction [8], and rational number [9]. The facts showed that the learning trajectory became an interesting topic to be further investigated in learning of mathematics.

Clements & Sarama [10] states that learning trajectory is a description of students' thinking and learning in particular mathematical domains and associated route guesses through a set of learning tasks designed to induce an action or mental process moving toward the progression of the thinking level. Based on the definition, there are two important aspects in learning trajectory, namely developmental progression and learning tasks. The developmental progression includes students' thinking and learning process in the mathematical domain. Students' thinking and learning process relate to a conjectured route on the benchmark of students' thinking levels through a hypothesised set of learning tasks to promote the development of a concept. Thus, learning trajectory can be defined as a student's thinking process in the process of learning created with the intention to achieve goals in certain mathematical domains.

Learning trajectory is based on a constructivist paradigm for the development of students' understanding. There are three components in a learning trajectory, namely the goal for students' learning, the learning activity, and students' thought and learning process [11]. The learning trajectory may not progress linearly. There are two types of learning trajectory, namely hypothetical learning trajectory and actual learning trajectory. The phrase of hypothetical learning trajectory was first



defined by Simon [11] as the teacher's prediction of the learning paths produced by students from less sophisticated to more sophisticated ideas and accumulated understandings. The characteristic of the hypothetical learning trajectory is flexible. Teachers may modify the learning trajectory that has been previously designed when students fail to understand the topic. The design of alternative learning trajectory can be used to help all students successfully achieving the learning goal. In the hypothetical learning trajectory, the actual learning trajectory is not knowable beforehand. Actual learning trajectory is obtained from implementing the hypothetical learning trajectory in the teaching experiment and analyzing students' learning path. Battista [12] focuses the idea of learning trajectory on the framework of a cognition-based assessment. This notion emphasizes the level of the model for a topic that not only describes a student's cognitive process, but also what things students can or cannot do, students' reasoning and conceptualization, a cognitive obstacle, and mental processes for progressing to higher levels.

Investigating students' learning trajectory in the mathematical learning process is crucial. The detection of the learning trajectory is the core of research projects, curricula, and professional development [10,11]. Learning trajectories have become an essential study in teaching and learning mathematics [10,11]. Knowing students' learning trajectory can assist educators in designing instructional model/strategy in response to students' mathematical thought process.

The construction task in geometry became one of the most studied issues in mathematics education [13], but there have been limited studies of students' learning trajectory for the triangle topic, especially triangle construction. Therefore, the purpose of this study is to investigate students' learning trajectory as the progressions of students' thinking about a triangle. The reason underlying the selection of the triangle topics because it is very widely used in everyday life. Yet, the triangle construction task becomes a problematic condition for some students. Students faced difficulty in constructing an equivalent triangle [14,15]. Graham & Chick [16] reported that some students were perplexed in determining all possible triangles that can be made from 20 matchsticks. Hence, teachers can use the result of this study to enhance the comprehension of students' learning progression about the triangle concept. This study makes a significant contribution related to students' learning trajectory. Teachers also can overcome students' misconceptions or mistakes.

## 2. Method

As a part of design research, this study focuses on students' learning trajectory. Design research is a design experiment that involves three phases: (1) preparing for the experiment (design), (2) implementing teaching experiment, and (3) carrying out retrospective analysis [17]. The design experiment aims to develop both learning processes theory and learning environment that supports the learning processes [17,18]. It is noteworthy that design research is not an experimental study. Design research is also labelled as developmental research [19] or design experiments [18].

Participants involved in this study were 22 students of 7<sup>th</sup> grade. They were selected purposively from one of Islamic Junior High School in Malang, Indonesia. The study used a qualitative approach. Data were collected through videotape, learning artifacts (student's work), and a classroom observation. In the teaching experiment, the second author was the teacher and the first author was the observer. We also focused on students' gesture and group discussion. Cobb et al. [18] emphasizes the usage of various data sources (e.g., gesture, social interaction, artifact, and classroom discourse) and technological support (e.g., audio-recording tools) for obtaining a good data.

In this study, we designed learning tasks to support instructional experiment and foster students' learning about a triangle. The teaching experiment process was recorded using a digital camera and was transcribed for detailed analysis. In addition, for data triangulation step, various types of data were collected including students' thinking about the triangle, field note, and student's work. We further transcribed the recording of the learning process, especially the conversation between students and teacher. Finally, we analyzed the data of students' learning trajectory.

### 3. Results and discussion

In the teaching experiment, the teacher divided the students into 5 groups with details of 3 groups (Group 1, Group 2 and Group 3) each of 4 students, and 2 groups (Group 4 and Group 5) each of 5 students. In the learning process, 4 other groups completed all the triangle construction tasks correctly, but there was one remaining group (Group 3) who answered with a construction duties D. All students in Group 3 determined 3 side lengths in Task D could not form a triangle. They did not realize that the 3 side lengths actually could form a triangle. Distribution of students' answer is described completely in Table 1.

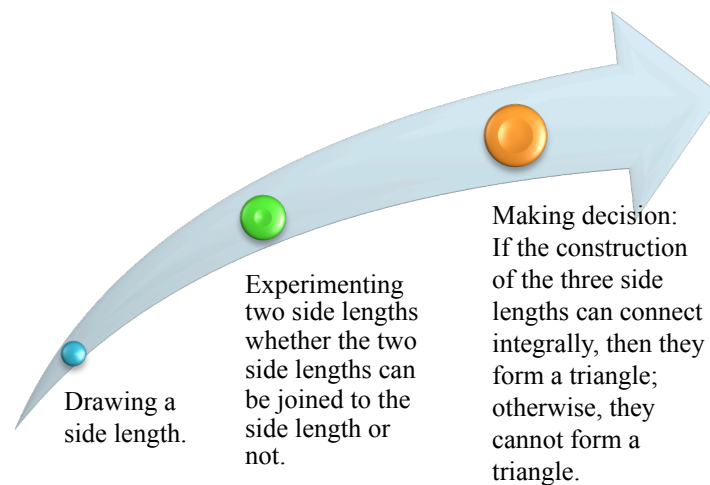
**Table 1.** Distribution of students' answer from each group

Construction task	Forming a triangle				
	Group 1	Group 2	Group 3	Group 4	Group 5
Task A (4 cm, 4.5 cm, and 6 cm)	Yes	Yes	Yes	Yes	Yes
Task B (3 cm, 2 cm, and 6 cm)	No	No	No	No	No
Task C (4 cm, 3 cm, and 4 cm)	Yes	Yes	Yes	Yes	Yes
Task D (3 cm, 4 cm, and 2 cm)	Yes	Yes	No	Yes	Yes
Task E (2 cm, 3 cm, and 5 cm)	No	No	No	No	No

In this article, the discussion about students' learning trajectory is represented by Group 2. The reason for choosing this group is because the students were very confident in expressing their ideas. They also cooperated solidly. They explored the construction tasks well. They made the first step by drawing a line segment. Then, they used two rulers to represent the other two line segments. They joined the two line segments to the line segment as shown in Figure 1. If 3 side lengths are connected to each other, they decided that it could form a triangle. However, if it does not happen, they decided that 3 side lengths did not form a triangle. Students' learning trajectory can be seen in Figure 2.



**Figure 1.** Students' activity in constructing a triangle



**Figure 2.** Student's learning trajectory in constructing a triangle

Students were enthusiastic in the discussion. In the construction task, each student gave a significant contribution. When students faced a problem/difficulty, they discussed again to find a solution. The teacher also gave some hints or questions to them. The following is the conversations protocol between teacher and students in group discussion.

- Teacher : How do you solve the task of triangle construction?
- Student 1: We used pencils, pens, and rulers. We drew a line segment first. We continued to use two rulers to check whether two line segments can be joined with one previous line segment. If it can connect, it means the triangle is formed. If it does not connect, yes...it cannot be made a triangle. Of the 5 tasks that have been done, Tasks A, C, and D could form a triangle, but Task B and E could not form a triangle.
- Teacher : How do you guarantee that the three side lengths can or cannot form a triangle?
- Student 1: Ya ... ya all line segments in Task A, C, and D connect each other forming a triangle when we construct the tasks. For Task B and E, if it is drawn it cannot form a triangle. Maybe like that, Sir. How are my arguments, friends?
- Student 2: Wait a minute ... Let's discuss this.
- Student 3: Yes, that's right.
- Student 4: But, how to solve the problem?
- Teacher : Let's pay attention to the problems. Noticing the task that can or cannot form a triangle. Is there a specific pattern?
- Student 2: (Student think for a long time) Oh ... ya we can investigate the group of triangle tasks that can form a triangle or not. There seems to be something unique on the sides of the triangle. Maybe we can find the pattern.
- Student 1: Aha... I found the idea. That is a good point, friend. Let's check all the side lengths. For side lengths 2 cm, 3 cm, and 5 cm (Task E), all possible experiments cannot form a triangle. For example, the length of the first line segment drawn is 5 cm. This is the 2cm side and 3cm side connected to the 5cm side, it is not possible to be a triangle. When both sides (2 cm and 3 cm) are joined, it is parallel to the 5 cm side. Next, for side lengths of 3 cm, 2 cm, and 6 cm (Task B), if 3 cm and 2 cm sides are joined, the total length is 5 cm. The total side is parallel to the third side and will not be able to form a triangle. The 3cm and 6cm sides are connected. When the result is tried to be joined with the third side, it still does not connect. For side lengths of 4 cm, 4.5 cm, and 6 cm (Task A), all combinations of two side lengths are always more than the third

side. Similarly, this is also valid for Task C and D.

Teacher : Wow... Great idea. So, what is the condition of the existence of a triangle?

Student 2: It is sought all possibilities, Sir. If all possibilities sum of two side lengths is more than the third side, then the triangle exists.

Student 1: It is definitely. The sum of any two side lengths has to greater than another side.

In another activity, a student in Group 2 used their index finger to represent two segments of a fused line. The student demonstrated the formation of a triangle whose sides have lengths 4 cm, 3 cm, and 4 cm. The student revealed that the triangle was anisosceles triangle. The studentconvincedthe teachers related to her argumentation and clarified her reasoning.The finding shows that the student used gestures to support and explain her reasoning. The fact is in accordance with the existing studies [20, 21] which reported that gestures have the contribution to students' reasoning.

In the process of learning, students in Group 2 initially had difficulty to determine the conclusion about the requirement of the forming a triangle given threeline segments. However, by the scaffolding of the teacher, they ccould ultimately make a conclusion that the sum of all possible two sides is more than the third side. The teacher also provided scaffolding to Group 3 regarding errors in Task D. The teacher instructed students to reconstruct TaskDcarefully. Scaffolding is very effective in overcoming thestudents' difficulties or errors. Some scholars stated that scaffolding can help students to gain success in problem-solving [22,23].

The success of teachers in providing scaffolding is inseparable from a good pedagogical content knowledge (PCK). PCK has a positive effect on students' understanding and learning process [24,25].In an effective instruction, teachers should not only master the material but also be able to apply appropriate learning strategies by using contingent dominants caffolding [26]. Teachers can previously design hypothetical learning trajectory, respond to events that occur in the classroom, overcome students' difficulties/misconceptions, and arouse students' thinking processes.

#### 4. Conclusion

To conclude, this study found students' learning trajectory about forming a triangle given three side lengths as shown in Figure 2. Students were able to determine whether the three side lengths can be made into a triangle or not. The learning process was significant because the design of instruction supported the invention of triangle theorem. Students discovered the theorem by the scaffolding of their teachers. Students also controlled their thinking when they faced a difficulty or a mistake. Therefore, in the instruction teachers can help students to realise, regulate, and evaluate their thought process as the component of metacognition in achieving learning goals. For further studies, it is imperative to examine students' metacognition and characteristics of teachers' scaffolding in the mathematics teaching.

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