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Preface

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Preface

One of factors that influences the growth of industry is the rapid and advanced enhancement of research on science and technology. There is a wealth of opportunities for science in industry, government, and in areas of applied science. This conference offers a forum for intellectual propositions and academic discussions on how science and technology could result in social prosperity through commercialization and industrialization.

Annual Conference of Science and Technology (Ancoset) was held by Universitas Kanjuruhan Malang. The conference aimed at showcasing the latest technological research and providing a forum in which to discuss insightful and innovative ideas on mathematics and science practice as well as facilitating debate, networking and professional development opportunities. The conference was conducted in Malang, Indonesia on August, 30th, 2018, and attended by more than 150 participants from mathematics, science and technology enthusiasts.

Five experts from Australia, Thailand, Malaysia and Indonesia contributed to give their knowledge and experiences on relevant themes. Audience came from several countries such as Indonesia, Malaysia and Australia. More than 130 papers were presented in the conference. With respect to more than 100 papers selected. The Ancoset committee herewith would congratulate the authors whose papers were published in this conference series. The committee hopes that all the findings and knowledge shared in this venture give beneficial impacts on the quality of mathematics and science.

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Dr. Asep Bayu Dani Nandiyanto
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Riza Weganofa, M.Pd

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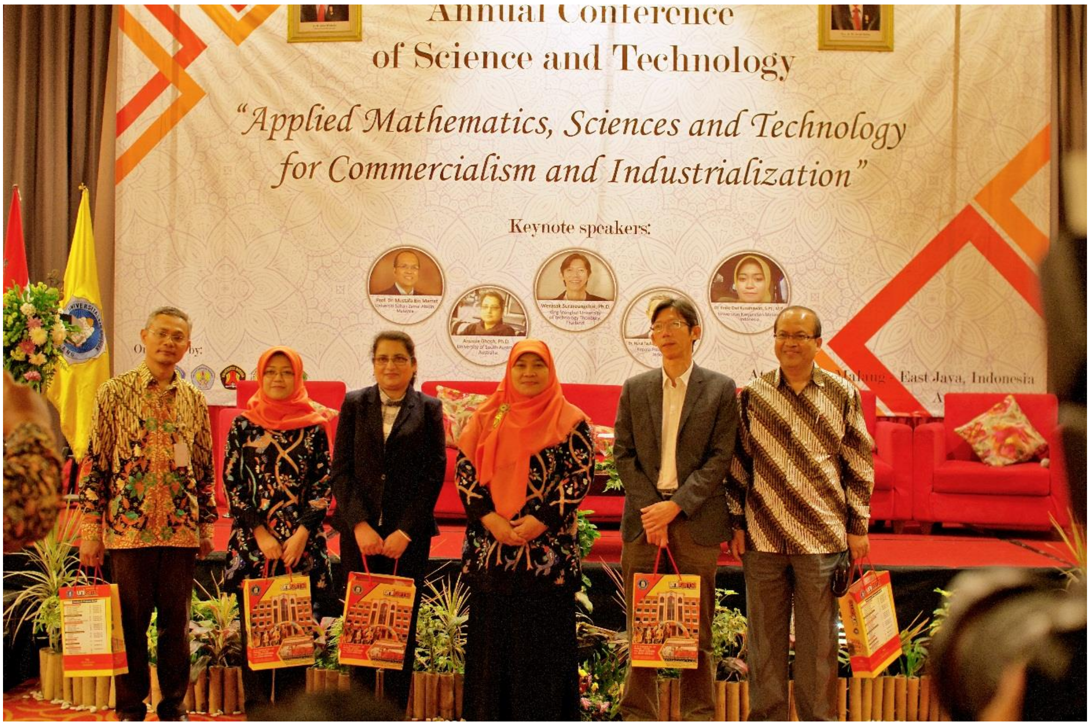
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Introduction
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The concept of eccentric distance sum index (EDSI) was introduced by Gupta, Singh and Madan [1] in 2002 as a new invariant graph which is a distance based topological descriptor. The concept of adjacent eccentric distance sum index (AEDSI) was introduced by Sardana and Madan [2] in 2002 as an adjacency cum distance based topological descriptor.


Domestication of Seluang (*Rasbora argyrotaenia*) from the Musi River Waters
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Eccentric distance sum and adjacent eccentric distance sum index of complement of subgroup graphs of dihedral group

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Eccentric distance sum and adjacent eccentric distance sum index of complement of subgroup graphs of dihedral group

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Abstract. Let $G = (V(G), E(G))$ is a connected simple graph. Let $ec(v)$ is the eccentricity of vertex v, $D(v) = \sum_{u \in V(G)} d(u, v)$ is the sum of all distances from vertex v and $deg(v)$ is the degree of vertex v in G. The eccentric distance sum index of G is defined as $\xi^{d}(G) = \sum_{v \in V(G)} ec(v)D(v)$ and the adjacent eccentric distance sum index of G is defined as $\xi^{sv}(G) = \sum_{v \in V(G)} \frac{ec(v)D(v)}{\deg(v)}$. For positive integer m and $m \geq 3$, let D_{2m} be dihedral group of order $2m$ and N is a normal subgroup of D_{2m}. The subgroup graph $\Gamma_N(D_{2m})$ of dihedral group D_{2m} is a simple graph with vertex set D_{2m} and two distinct vertices x and y are adjacent if and only if $xy \in N$. In the present paper, we compute eccentric distance sum and adjacent eccentric distance sum index of complement of subgroup graph of dihedral group D_{2m}. Total eccentricity, eccentric connectivity index, first Zagreb index, and second Zagreb index of these graphs are also determined.

1. Introduction

The concept of eccentric distance sum index (EDSI) of a connected graph was first introduced by Gupta, Singh, and Madan [1] in 2002. In the same year, Sardana and Madan [2] introduced the concept of adjacent eccentric distance sum index (AEDSI) of a graph. In the beginning, research on EDSI and AEDSI was closely related to the chemical properties and biological activities of molecular structures [1-3]. In subsequent developments, research on EDSI and AEDSI is not only limited to graphs relating to molecular structures. Until now, EDSI and AEDSI have become research topics that are widely studied by researchers on various graphs [4-18].

New concepts of graphs are increasingly being introduced including graphs obtained from an algebraic structure such as group or ring. One of them is the subgroup graph introduced by Anderson, Fasteen, and Lagrange [19] in 2012. If G is a finite group and N is a normal subgroup of G, then the subgroup graph $\Gamma_N(G)$ of G is a simple and undirected graph with $V(\Gamma_N(G)) = G$ and $xy \in E(\Gamma_N(G))$ if and only if $xy \in N$. This implies that $\overline{\Gamma_N(G)}$ also a simple and undirected graph [20].

In this paper, we determined the EDSI and AEDSI of complement of subgroup graphs of dihedral group. The discussion is specific to the complements of subgroup graphs of dihedral group because the discussed subgroup graphs are not connected.

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2. Literature review

Suppose graph *G* is a simple and connected of order $|\boldsymbol{V}(\boldsymbol{G})| = \boldsymbol{p}$ and size $|\boldsymbol{E}(\boldsymbol{G})| = \boldsymbol{q}$. Let $\boldsymbol{N}(\boldsymbol{u})$ denoted the neighbor of a vertex *u* in *G*. The degree of a vertex *u* in *G* is defined as $\boldsymbol{deg}(\boldsymbol{u}) = |\boldsymbol{N}(\boldsymbol{u})|$. The distance $\boldsymbol{d}(\boldsymbol{v}, \boldsymbol{w})$ between two vertices *v* and *w* in *G* is defined as the length of the shortest *v-w* path in *G*. The sum of all distances from vertex *v* to any vertices of *G* is denoted by $\boldsymbol{D}(\boldsymbol{v})$. Therefore, $\boldsymbol{D}(\boldsymbol{v}) = \sum_{\boldsymbol{w}\in \boldsymbol{V}(\boldsymbol{G})} \boldsymbol{d}(\boldsymbol{v}, \boldsymbol{w})$. The eccentricity $\boldsymbol{ec}(\boldsymbol{v})$ of vertex *v* is defined by $\boldsymbol{ec}(\boldsymbol{v}) = \mathbf{max}\{\boldsymbol{d}(\boldsymbol{v}, \boldsymbol{w}) : \boldsymbol{w} \in \boldsymbol{V}(\boldsymbol{G})\}$. Radius $\boldsymbol{rad}(\boldsymbol{G})$ of graph *G* is defined by $\boldsymbol{rad}(\boldsymbol{G}) = \mathbf{min}\{\boldsymbol{ec}(\boldsymbol{v}) : \boldsymbol{v} \in \boldsymbol{V}(\boldsymbol{G})\}$ while diameter $\boldsymbol{diam}(\boldsymbol{G})$ of graph *G* is defined by $\boldsymbol{diam}(\boldsymbol{G}) = \mathbf{max}\{\boldsymbol{ec}(\boldsymbol{v}) : \boldsymbol{v} \in \boldsymbol{V}(\boldsymbol{G})\}$ [21]. The total eccentricity of graph *G* is defined as

$$\boldsymbol{\xi}(\boldsymbol{G}) = \sum_{\boldsymbol{v}\in \boldsymbol{V}(\boldsymbol{G})} \boldsymbol{ec}(\boldsymbol{v}).$$

The eccentric distance sum index (EDSI) of a connected graph *G* was first introduced by Gupta, Singh, and Madan [1] in 2002 and defined as

$$\xi^d(G) = \sum_{v\in V(G)} ec(v)D(v).$$

In the same year, Sardana and Madan [2] introduced the concept of adjacent eccentric distance sum index (AEDSI) and defined as

$$\xi^{sv}(G) = \sum_{v\in V(G)} \frac{ec(v)D(v)}{deg(v)}.$$

Other indices based on eccentricity of vertex are eccentric connectivity index, first Zagreb index, and second Zagreb index. Sharma, Goswami, and Madan [22] defined the eccentric connectivity index (ECI) of a connected graph as

$$\xi^c(G) = \sum_{v\in V(G)} ec(v)\deg(v).$$

The first and second Zagreb indices of a connected graph were first introduced by Gutman and Trinajstić [23] and revised by Ghorbani and Hosseinzadeh [24]. For a connected graph *G*, the first Zagreb index is defined as

$$E_1(G) = \sum_{v\in V(G)} (ec(v))^2,$$

while the second Zagreb index is defined as

$$E_2(G) = \sum_{vw\in E(G)} ec(v)ec(w).$$

Let $D_{2m} = \langle r, s\rangle$ is dihedral group of order 2*m* where *m* is positive integer and *m* ≥ 3. If *m* is odd, the normal subgroup of D_{2m} are $\langle 1\rangle$, $\langle r^d\rangle$ where *d* divides *m* and D_{2n} itself. If *m* is even, the normal subgroups of D_{2m} are $\langle 1\rangle$, $\langle r^d\rangle$ where *d* divides *m*, $\langle r^2, s\rangle$, $\langle r^2, rs\rangle$ and D_{2m} itself [25]. By taking any normal subgroup *N* of D_{2m}, then subgroup graph $\Gamma_N(D_{2m})$ is an undirected simple graph. When $N = D_{2m}$ we have $\Gamma_{D_{2m}}(D_{2m}) = K_{2m}$. So, $ec(v) = 1$, $deg(v) = 2m - 1$ and $D(v) = 2m - 1$ for any vertex v in $\Gamma_{D_{2m}}(D_{2m})$. By direct computation, we obtain

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$$\xi^c\left(\Gamma_{D_{2m}}(D_{2m})\right) = \xi^d\left(\Gamma_{D_{2m}}(D_{2m})\right) = 2m(2m-1),$$
$$\xi\left(\Gamma_{D_{2m}}(D_{2m})\right) = \xi^{sv}\left(\Gamma_{D_{2m}}(D_{2m})\right) = 2m,$$
$$E_1\left(\Gamma_{D_{2m}}(D_{2m})\right) = 2m,$$

and

$$E_2\left(\Gamma_{D_{2m}}(D_{2m})\right) = m(2m-1).$$

Next, we present our results on the subgroup graphs of the dihedral group for other normal subgroups.

3. Method

To determine the eccentric distance sum and adjacent eccentric distance sum index of the complement of the subgroup graph of dihedral groups, several steps taken in this study are the following.

- Drawing the subgroup graph of dihedral group D_{2m} for m = 3, 4, 5, 6, 7, 8.
- Drawing the complement of each subgroup graph in step (1).
- Calculating the eccentric distance sum and adjacent eccentric distance sum index of each complement of the subgroup graph in step (2).
- Formulating conjectures based on the eccentric distance sum and adjacent eccentric distance sum index pattern.
- Reformulating the conjectures as theorems and giving their formal proof.

4. Results

The normal subgroup $\langle r\rangle$ of D_{2m} is $\langle r\rangle = \{1, r, r^2, \dots, r^{m-1}\}$. We have $\Gamma_{\langle r\rangle}(D_{2m})$ is a disconnected graph and $\Gamma_{\langle r\rangle}(D_{2m}) = 2K_m$. The vertex sets of each component are $\{1, r, r^2, \dots, r^{m-1}\}$ and $\{s, sr, sr^2, \dots, sr^{m-1}\}$. So, $\overline{\Gamma_{\langle r\rangle}(D_{2m})} = K_{m,m}$. Then, $deg(v) = m$, $ec(v) = 2$ and $D(v) = 3m - 2$ for all $v \in V\left(\overline{\Gamma_{\langle r\rangle}(D_{2m})}\right)$. We obtain the following results by direct computation.

Theorem 3.1 *For* $m \geq 3$, then

(a) $\xi\left(\overline{\Gamma_{\langle r\rangle}(D_{2m})}\right) = 4m$,
(b) $\xi^c\left(\overline{\Gamma_{\langle r\rangle}(D_{2m})}\right) = 4m^2$,
(c) $\xi^d\left(\overline{\Gamma_{\langle r\rangle}(D_{2m})}\right) = 12m^2 - 8m$,
(d) $\xi^{sv}\left(\overline{\Gamma_{\langle r\rangle}(D_{2m})}\right) = 12m - 8$,
(e) $E_1\left(\overline{\Gamma_{\langle r\rangle}(D_{2m})}\right) = 8m$, and
(f) $E_2\left(\overline{\Gamma_{\langle r\rangle}(D_{2m})}\right) = 4m^2$.

Proof

(a) $\xi\left(\overline{\Gamma_{\langle r\rangle}(D_{2m})}\right) = \sum_{v\in V\left(\overline{\Gamma_{\langle r\rangle}(D_{2m})}\right)} ec(v) = \sum_{v\in V\left(\overline{\Gamma_{\langle r\rangle}(D_{2m})}\right)} 2 = 2(2m) = 4m.$
(b) $\xi^c\left(\overline{\Gamma_{\langle r\rangle}(D_{2m})}\right) = \sum_{v\in V\left(\overline{\Gamma_{\langle r\rangle}(D_{2m})}\right)} ec(v)deg(v) = \sum_{v\in V\left(\overline{\Gamma_{\langle r\rangle}(D_{2m})}\right)} 2m = 2m(2m) = 4m^2.$
(c) $$\begin{aligned}\xi^d\left(\overline{\Gamma_{\langle r\rangle}(D_{2m})}\right) &= \sum_{v\in V\left(\overline{\Gamma_{\langle r\rangle}(D_{2m})}\right)} ec(v)D(v) \\ &= \sum_{v\in V\left(\overline{\Gamma_{\langle r\rangle}(D_{2m})}\right)} 2(3m-2) \\ &= 2m[2(3m-2)] \\ &= 12m^2 - 8m.\end{aligned}$$
(d) $\xi^{sv}\left(\overline{\Gamma_{\langle r\rangle}(D_{2m})}\right) = \sum_{v\in V\left(\overline{\Gamma_{\langle r\rangle}(D_{2m})}\right)} \frac{ec(v)D(v)}{deg(v)}$

$$= \sum_{v \in V\left(\overline{\Gamma_{\langle r \rangle}(D_{2m})}\right)} \frac{2(3m-2)}{m}$$
$$= 2m\left(\frac{2(3m-2)}{m}\right)$$
$$= 12m - 8.$$

(e) $E_1\left(\overline{\Gamma_{\langle r \rangle}(D_{2m})}\right) = \sum_{v \in V\left(\overline{\Gamma_{\langle r \rangle}(D_{2m})}\right)} (ec(v))^2 = \sum_{v \in V\left(\overline{\Gamma_{\langle r \rangle}(D_{2m})}\right)} 2^2 = 2m(4) = 8m.$

(f) $E_2\left(\overline{\Gamma_{\langle r \rangle}(D_{2m})}\right) = \sum_{uv \in E\left(\overline{\Gamma_{\langle r \rangle}(D_{2m})}\right)} ec(u)ec(v) = m^2 \cdot 2 \cdot 2 = 4m^2.$ □

The following are results for $m \geq 4$ and m is even.

Theorem 3.2 *For* $m \geq 4$ and m is even, then

(a) $\xi\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right) = 4m,$

(b) $\xi^c\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right) = 6m^2,$

(c) $\xi^d\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right) = 10m^2 - 8m,$

(d) $\xi^{sv}\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right) = \frac{20m-16}{3},$

(e) $E_1\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right) = 8m$, and

(f) $E_2\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right) = 12m.$

Proof

For $m \geq 4$ and m is even, $\langle r^2 \rangle = \{1, r^2, r^4, \ldots, r^{m-2}\}$ is a normal subgroup. So, the subgroup graph $\Gamma_{\langle r^2 \rangle}(D_{2m}) = 4K_{m/2}$.Then, $\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})} = K_{m/2,m/2,m/2,m/2}$. We obtain $deg(v) = 3m/2$, $ec(v) = 2$, and $D(v) = (5m-4)/2$ for all $v \in V\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right)$. Therefore

(a) $\xi\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right) = \sum_{v \in V\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right)} ec(v) = \sum_{v \in V\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right)} 2 = 2(2m) = 4m.$

(b) $\xi^c\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right) = \sum_{v \in V\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right)} ec(v)deg(v) = \sum_{v \in V\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right)} 2 \cdot \frac{3m}{2} = 2m(3m) = 6m^2.$

(c) $\xi^d\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right) = \sum_{v \in V\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right)} ec(v)D(v)$
$$= \sum_{v \in V\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right)} 2 \cdot \frac{5m-4}{2}$$
$$= 2m(5m-4)$$
$$= 10m^2 - 8m.$$

(d) $\xi^{sv}\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right) = \sum_{v \in V\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right)} \frac{ec(v)D(v)}{deg(v)}$
$$= \sum_{v \in V\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right)} \frac{2 \cdot \frac{5m-4}{2}}{\frac{3m}{2}}$$
$$= 2m\left(\frac{2(5m-4)}{3m}\right)$$
$$= \frac{20m^2 - 16m}{3m}$$
$$= \frac{20m-16}{3}.$$

(e) $E_1\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right) = \sum_{v \in V\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right)} (ec(v))^2 = \sum_{v \in V\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right)} 2^2 = 2m(4) = 8m.$

(f) $E_2\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right) = \sum_{uv \in E\left(\overline{\Gamma_{\langle r^2 \rangle}(D_{2m})}\right)} ec(u)ec(v) = \frac{6m}{2} \cdot 2 \cdot 2 = 12m.$ □

For $m \geq 4$ and m is even, the normal subgroup $\langle r^2, s \rangle$ of dihedral group D_{2m} is $\langle r^2, s \rangle = \{1, r^2, r^4, \ldots, r^{m-2}, s, sr^2, sr^4, \ldots, sr^{m-2}\}$. So, $\Gamma_{\langle r^2, s \rangle}(D_{2m}) = 2K_m$ and we have $\overline{\Gamma_{\langle r^2, s \rangle}(D_{2m})} = K_{m,m}$. Then, $deg(v) = m$, $ec(v) = 2$, and $D(v) = 3m - 2$ for all $v \in V\left(\overline{\Gamma_{\langle r^2, s \rangle}(D_{2m})}\right)$. Hence, we have the following results.

Theorem 3.3 *For* $m \geq 4$ and m is even, then

(a) $\xi\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2m})}\right) = 4m$,
(b) $\xi^c\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2m})}\right) = 4m^2$,
(c) $\xi^d\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2m})}\right) = 12m^2 - 8m$,
(d) $\xi^{sv}\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2m})}\right) = 12m - 8$,
(e) $E_1\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2m})}\right) = 8m$, and
(f) $E_2\left(\overline{\Gamma_{\langle r^2,s\rangle}(D_{2m})}\right) = 4m^2$.

We can easily observe that $\Gamma_{\langle r^2,rs\rangle}(D_{2m}) = 2K_m$, for $m \geq 4$ and m is even. Then, $\overline{\Gamma_{\langle r^2,rs\rangle}(D_{2m})} = K_{m,m}$. This fact brings us to the following results.

Theorem 3.4 For $m \geq 4$ and m is even, then

(a) $\xi\left(\overline{\Gamma_{\langle r^2,rs\rangle}(D_{2m})}\right) = 4m$.
(b) $\xi^c\left(\overline{\Gamma_{\langle r^2,rs\rangle}(D_{2m})}\right) = 4m^2$.
(c) $\xi^d\left(\overline{\Gamma_{\langle r^2,rs\rangle}(D_{2m})}\right) = 12m^2 - 8m$.
(d) $\xi^{sv}\left(\overline{\Gamma_{\langle r^2,rs\rangle}(D_{2m})}\right) = 12m - 8$.
(e) $E_1\left(\overline{\Gamma_{\langle r^2,rs\rangle}(D_{2m})}\right) = 8m$.
(f) $E_2\left(\overline{\Gamma_{\langle r^2,rs\rangle}(D_{2m})}\right) = 4m^2$.

The proof of Theorem 3.3 and 3.4 are similar to the proof of Theorem 3.1.

5. Conclusion

We have determined the formula of total eccentricity, eccentric connectivity index, eccentric distance sum index, adjacent eccentric distance sum index, first Zagreb index, and second Zagreb index of some subgroup graphs of dihedral groups. The eccentric distance sum index and adjacent eccentric distance sum index of other subgroup graphs of dihedral group that are not discussed in this paper still need to be computed in further research.

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