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Determination of consumer price index with generalized space-time autoregressive

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Abstract. Customer Price Index is one way to analyze the level of consumption needs of people for goods and services within a certain time. The development of the current Customer Price Index determination besides being influenced by time is also influenced by spatial heterogeneity between regions. In this study, the determination of the Customer Price Index was carried out in three cities in Probolinggo, Surabaya, and Kediri using the Generalized Space-Time Autoregressive (GSTAR) approach. The GSTAR model is a time-series data analysis model that shows the existence of spatial heterogeneity between time and between regions. Based on the test results of the Matrix Autocorrelation Function and the Matrix Partial Autocorrelation Function, the best model is GSTAR with order 1 with the smallest Akaike Information Criterion value found in lag 1 which is 90.924.

1. Introduction

Customer Price Index is an indicator of economic change measured by the amount/index of the development of the price of goods/services consumed by consumers/society at a certain time [1]. Analysis of the current level of consumer prices in the community is of particular interest for researchers, especially in observing Customer Price Index behavior patterns resulting in spatial heterogeneity between time and location.

As an illustration of the East Java Customer Price Index, especially in the City of Probolinggo, the City of Surabaya, and the City of Kediri the data patterns indicate the presence of elements of spatial heterogeneity and interrelation between one location and another location. To facilitate calculations, in this study the Customer Price Index data from Probolinggo City, Surabaya City, and Kediri City are modeled using the Generalized Space-Time Autoregressive (GSTAR) approach.

The GSTAR model is a generalization of the Space-Time Autoregressive (STAR) model introduced by [2]. The GSTAR model is used for time series data analysis that shows the existence of spatial heterogeneity between time and location in multivariate data. In the GSTAR model assumes that the error is a white noise process with the mean of zero and constant variance [3].

The GSTAR model is a generalization of the Space-Time Autoregressive (STAR) which shows the spatial heterogeneity between time and location. In the GSTAR model the parameter value Φ_{kl} is heterogeneous between locations and observation times. So that the parameter values in the GSTAR model may vary at each location.

The general form of the GSTAR model with the order (p) dan spatial order $\lambda_1, \lambda_2, \dots, \lambda_p$ with parameter values Φ_{kl} differ between locations of observation [4]:



$$\mathbf{Z}(t) = \sum_{i=1}^p \left[\Phi_{k0} + \sum_{j=1}^{\lambda_k} \Phi_{i1} \mathbf{W} \right] \mathbf{Z}(t-i) + \mathbf{e}(t) \tag{1}$$

where:

p: order autoregressive time.

λ_k : spatial order of the *autoregressive term* to- k .

Φ : autoregressive parameters at time lag k dan spatial lag i

\mathbf{W} : $N \times N$ size weight matrix for spatial order i (where $i = 0, 1, \dots, N$)

$\mathbf{Z}(t)$: random vector of size $N \times 1$ at the t -time.

$\mathbf{e}(t)$: error vector at time t of size $N \times 1$, assumed to be distributed normally with a mean of Zero ($E[\mathbf{Z}(t) = \mathbf{0}]$) and constant variance.

The identification of the model to check the suitability of the GSTAR model has been discussed by [5].

In general, the GSTAR model with the order autoregressive p and spatial order $\lambda_1, \lambda_2, \dots, \lambda_p$ can be expressed as a matrix [6]

$$\begin{pmatrix} Z_1(t) \\ Z_2(t) \\ \vdots \\ Z_n(t) \end{pmatrix} = \begin{pmatrix} \Phi_{s0}^{(1)} & 0 & \dots & 0 \\ 0 & \Phi_{s0}^{(2)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \Phi_{s0}^{(n)} \end{pmatrix} \begin{pmatrix} Z_1(t-s) \\ Z_2(t-s) \\ \vdots \\ Z_n(t-s) \end{pmatrix} + \begin{pmatrix} \Phi_{sk}^{(1)} & 0 & \dots & 0 \\ 0 & \Phi_{sk}^{(2)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \Phi_{sk}^{(n)} \end{pmatrix} x + \begin{pmatrix} 0 & W_{12} & \dots & W_{1n} \\ W_{21} & 0 & \dots & W_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ W_{n1} & W_{n2} & \dots & 0 \end{pmatrix} \begin{pmatrix} Z_1(t-s) \\ Z_2(t-s) \\ \vdots \\ Z_n(t-s) \end{pmatrix} + \begin{pmatrix} e_1(t) \\ e_2(t) \\ \vdots \\ e_n(t) \end{pmatrix} \tag{2}$$

by substituting $V_i(t) = \sum_{j=1}^n W_{ij} Z_j(t)$ into the equation (2) :

$$\begin{pmatrix} Z_1(t) \\ Z_2(t) \\ \vdots \\ Z_n(t) \end{pmatrix} = \begin{pmatrix} \Phi_{s0}^{(1)} & 0 & \dots & 0 \\ 0 & \Phi_{s0}^{(2)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \Phi_{s0}^{(n)} \end{pmatrix} \begin{pmatrix} Z_1(t-s) \\ Z_2(t-s) \\ \vdots \\ Z_n(t-s) \end{pmatrix} + \begin{pmatrix} \Phi_{sk}^{(1)} & 0 & \dots & 0 \\ 0 & \Phi_{sk}^{(2)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \Phi_{sk}^{(n)} \end{pmatrix} \begin{pmatrix} V_1(t-s) \\ V_2(t-s) \\ \vdots \\ V_n(t-s) \end{pmatrix} + \begin{pmatrix} e_1(t) \\ e_2(t) \\ \vdots \\ e_n(t) \end{pmatrix} \tag{3}$$

where :

$\mathbf{Z}(t) = Z_1(t), Z_2(t), \dots, Z_n(t)$

and

$$w_{ij}^{(0)} = \begin{cases} 1, & \text{for } i = j \\ 0, & \text{for } i \neq j \end{cases}$$

2. Methods

The data used in this study are the secondary data Customer Price Index of Probolinggo City, Surabaya City and Kediri City from 2009 to 2018 [7]. Furthermore, the data is analyzed with the following stages of research (Figure 1):

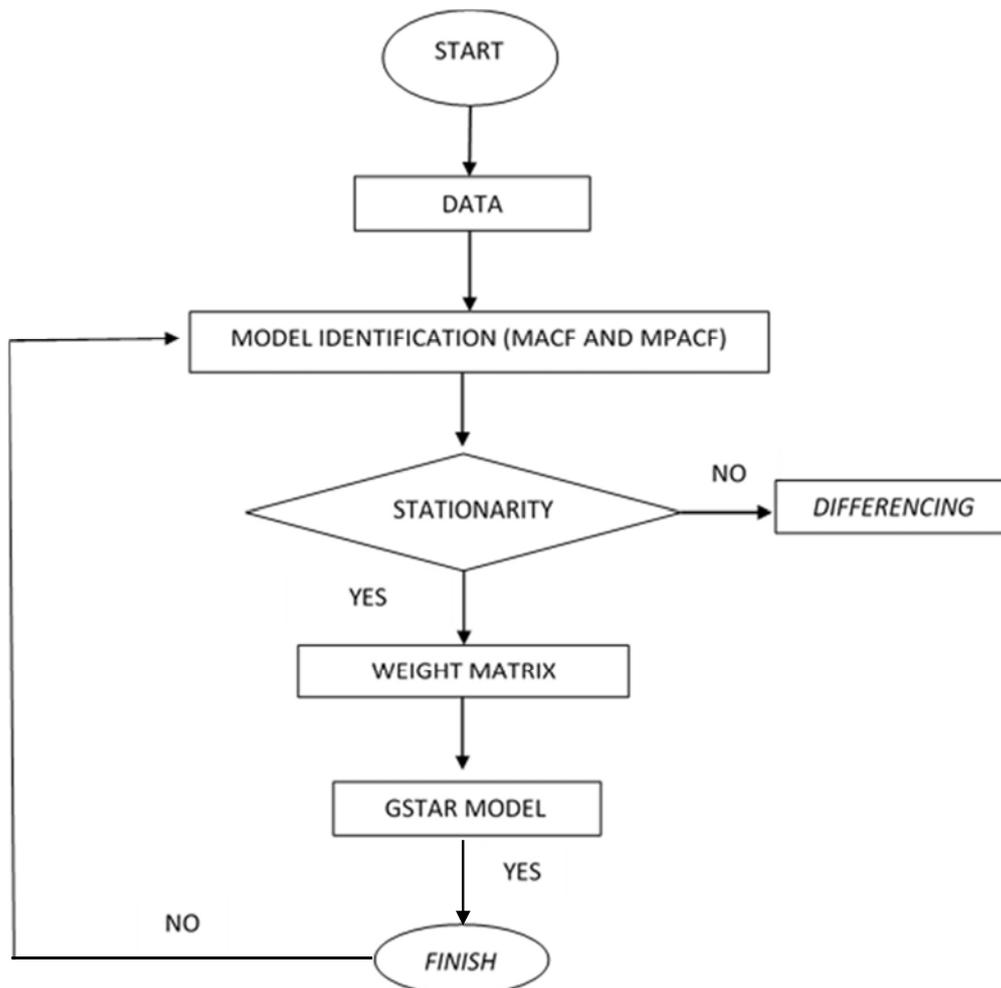


Figure 1. The Step of the research.

3. Result and discussion

3.1. Data Description

The Customer Price Index data used in this study have the same time-frame autoregressive pattern from 2009 to 2018 with the Customer Price Index plot as follows Figure 2. Figure 2 shows the fluctuation of Customer Price Index data for the three cities with the same pattern (trend), where economic growth has increased from year to year. Because the Customer Price Index plot of the three cities has the same pattern, it is assumed that there is an influence between the time and location of the study.

Based on Table 1. with a significance level of α of 5%, the Customer Price Index correlation value of Probolinggo City, Surabaya City, and Kediri City has a relatively similar Customer Price Index correlation value, where the probability value between the three locations is smaller than the significant level of α . So it can be concluded that between locations have a very strong correlation.

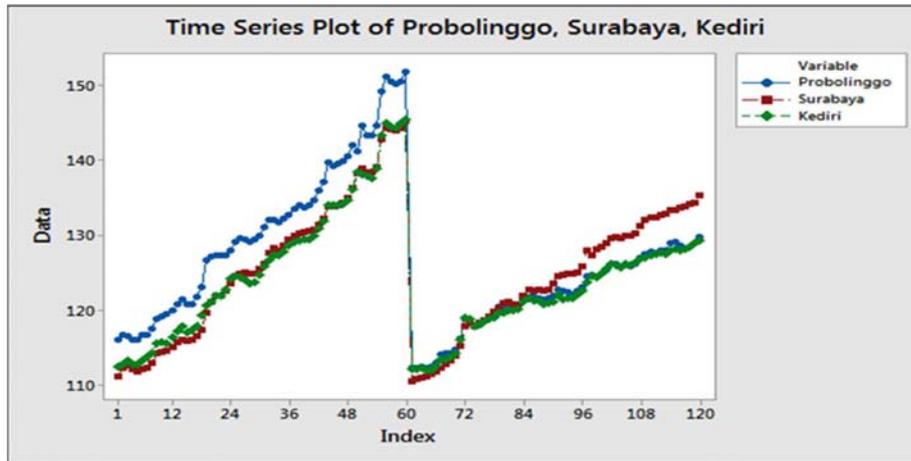


Figure 2. Simultaneous Customer Price Index Time Series Data Plots.

Table 1. Correlation Value of Customer Price Index.

Location	Z1 (Probolinggo)	Z2 (Surabaya)	Z3 (Kediri)
Z1 (Probolinggo)	1	0.916	0.977
p-value		(< 0.001)	(< 0.001)
Z2 (Surabaya)	0.916	1	0.974
p-value	(< 0.001)		(< 0.001)
Z3 (Kediri)	0.977	0.974	1
p-value	(< 0.001)	(< 0.001)	

3.2. Identification of the GSTAR Customer Price Index Model

After obtaining the homogeneity of the correlation value, then the GSTAR model Customer Price Index stationary was identified using spatial order 1. The identification of this model uses the Augmented Dikey-Fuller (ADF) unit root test with the MACF and MPACF models.

Based on Table 2. Customer Price Index in Probolinggo City, Surabaya City, and Kediri City shows a non-stationary pattern, where the probability value (p-value) is greater than the significant level α . Therefore, it can be concluded that the Customer Price Index in Probolinggo City, Surabaya City, and Kediri City is not stationary because of the influence of autocorrelation between locations and times. To eliminate the existence of autocorrelation by first differencing the model using the Augmented Dikey-Fuller unit root test.

Table 3 explains that the Customer Price Index in Probolinggo City, Surabaya City, and Kediri City shows the probability value (p-value) is smaller than the significant level, which means that the data is stationary and does not contain spatial autocorrelation.

Based on Figure 3 the correlation results of the GSTAR model on the Customer Price Index data indicate a positive partial autocorrelation. Which means that there is a spatial heterogeneity between Probolinggo City, Surabaya City, and Kediri City. Based on the results of Figure 4. The obtained Partial Autocorrelation Function (MPACF) matrix shows stationary data patterns after the 1st lag. This means that the GSTAR model can be used as a determinant of the Customer Price Index of Probolinggo City, Surabaya City, and Kediri City.

Table 2. Test ADF with MACF and MPACF Models.

Location	t-Statistics	t-Table ($\alpha = 0.05$)	Probability	Information
Probolinggo	-2.313	-3.448	0.424	Nonstasioner
Surabaya	-2.238	-3.448	0.464	Nonstasioner
Kediri	-2.310	-3.448	0.425	Nonstasioner

Table 3. Test the ADF Consumer Price Index Differencing data first.

Location	t-Statistics	t-Table ($\alpha = 0.05$)	Probability	Information
Probolinggo	-10.874	-3.448	0.000	Stasioner
Surabaya	-10.748	-3.448	0.000	Stasioner
Kediri	-10.588	-3.448	0.000	Stasioner

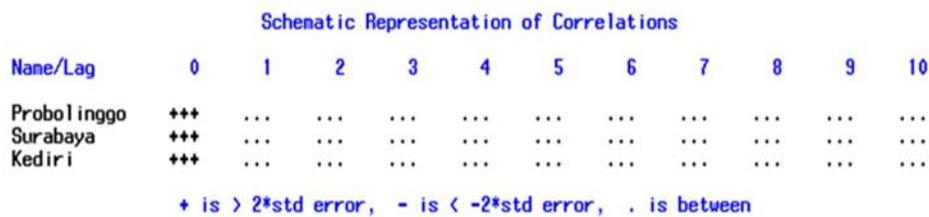


Figure 3. Simultaneous Customer Price Index MCF Model data.

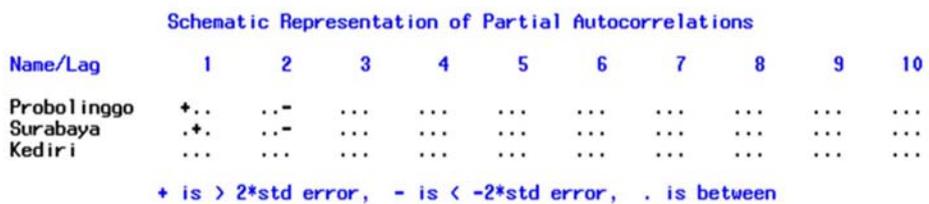


Figure 4. Simultaneous Customer Price Index MPACF data scheme.

3.3. Determination of the Consumer Price Index with the G STAR

After obtaining the stationary data for the mean and variance, the next step is to determine the Customer Price Index using the G STAR model by determining the order and estimation parameters using the inverse distance weight. Determination of the order of the G STAR model by determining the optimal lag based on the value of the Akaike Information Criterion.

Based on Figure 5, the smallest Akaike Information Criterion value is in lag 1 which is 90,924. So it can be stated that the Customer Price Index formed is the G STAR model with order 1 or G STAR (1). After obtaining the Customer Price Index model G STAR (1), the weighting is then searched using the inverse distance matrix. The inverse weight of the distance will give a value according to the distance between regions. The farther the distance between regions, the smaller the value of the resulting weight.

The STATESPACE Procedure										
Information Criterion for Autoregressive Models										
Lag=0	Lag=1	Lag=2	Lag=3	Lag=4	Lag=5	Lag=6	Lag=7	Lag=8	Lag=9	Lag=10
-81.5816	-90.9242	-77.1005	-62.1902	-51.6606	-37.7895	-35.6159	-27.7896	-16.5046	-5.54466	-36.7337

Figure 5. Simultaneous Value of Information Criterion Information.

$$\begin{aligned}
 \begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix} &= \begin{bmatrix} W_{11} & W_{12} & W_{13} \\ W_{21} & W_{22} & W_{23} \\ W_{31} & W_{32} & W_{33} \end{bmatrix} \\
 \begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix} &= \begin{bmatrix} 0 & W_{12} & W_{13} \\ W_{21} & 0 & W_{23} \\ W_{31} & W_{32} & 0 \end{bmatrix} \\
 \begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix} &= \begin{bmatrix} 0 & \frac{107}{206+124} & \frac{206}{107+124} \\ \frac{107}{206+124} & 0 & \frac{124}{107+206} \\ \frac{206}{107+124} & \frac{124}{107+206} & 0 \end{bmatrix} \tag{4} \\
 &= \begin{bmatrix} 0 & 0.324 & 0.892 \\ 0.324 & 0 & 0.396 \\ 0.892 & 0.396 & 0 \end{bmatrix}
 \end{aligned}$$

After obtaining the weighting value, using equation (3) and the parameter value Φ_{kl} is heterogeneous between the time and location of observation, then the Customer Price Index can be determined with the GSTAR Model (1) in Probolinggo City, Surabaya City, and Kediri City.

i. Probolinggo City Customer Price Index Model

$$Z(Probolinggo, t) = \Phi_{10} Z(Probolinggo, t - 1) + \Phi_{12} (0.324)Z(Surabaya, t - 1) + \Phi_{13} (0.892)Z(Kediri, t - 1) + e(Probolinggo, t)$$

ii. Surabaya City Customer Price Index Model

$$Z(Surabaya, t) = \Phi_{20} Z(Probolinggo, t - 1) + \Phi_{21} (0.324)Z(Surabaya, t - 1) + \Phi_{23} (0.396)Z(Kediri, t - 1) + e(Surabaya, t)$$

iii. Kediri City Customer Price Index Model

$$Z(Kediri, t) = \Phi_{30} Z(Probolinggo, t - 1) + \Phi_{31} (0.892)Z(Kediri, t - 1) + \Phi_{32} (0.396)Z(Surabaya, t - 1) + e(Kediri, t)$$

The Customer Price Index equation with the GSTAR Model (1) is the best model if it meets the White Noise assumption and is a multivariate normal distribution.

Figure 6 shows that the AIC values are in the AR (0) and MA (0) lag, which means that the residuals of the three equations above are independent and there is a positive partial autocorrelation of the Customer Price Index between Probolinggo City, Surabaya City, and Kediri City.

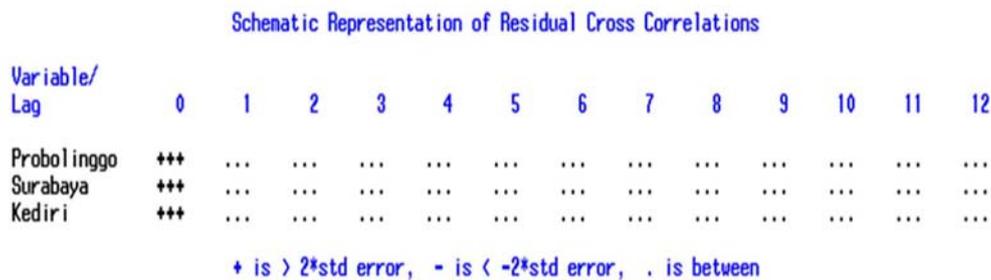


Figure 6. Plot Cross-Correlation of Residuals.

4. Conclusion

Based on the results of the Customer Price Index modeling in Probolinggo City, Surabaya City, and Kediri City, the best model is GSTAR (1). By using distance inverse matrix weighting produced residuals that meet the assumption of white noise and normal multivariate. So that the GSTAR model (1) can be used to determine the Customer Price Index in Probolinggo City, Surabaya City, and Kediri City.

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