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Anti-adjacency and Laplacian spectra of inverse graph of group of integers modulo *n*

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Abstract. Research on the spectra of a graph still attracts the attention of many researchers over the last decades. In addition, research related to graphs obtained from an algebraic structure such as groups and rings is also growing. This paper determines the spectrum of the anti-adjacency and Laplacian matrices of inverse graph of a finite commutative group, namely the addition group of integers modulo *n*. It can be concluded that all eigenvalues of anti-adjacency and Laplacian matrices of the inverse graph of addition group of integers modulo *n* are integer

1. Introduction

Since Norman Bigg [1] first introduced the spectrum adjacency or spectrum concept of a graph in 1974, researchers continue to develop various other spectrum concepts. At present, several spectrum concepts have been developed and studied, such as Laplacian [2-9], signless Laplacian [10-16], detour [17], distance [18-23], distance Laplacian [24], distance signless Laplacian [25], detour distance Laplacian [26], color Laplacian [27] and color signless Laplacian [28] spectra of various types of graphs. This shows that the spectrum topic of a graph is still in great demand by researchers. Laplacian spectrum and its variation received more attention than the adjacency spectrum and detour spectrum from the researchers.

On the other hand, the study of graphs obtained from an algebraic structure also continues to develop and produces various types of graphs. Some examples of graphs obtained from a group are Cayley graph [29], subgroup graph [30], commuting graph [31,32], non-commuting graph [33], identity graph [29], graph conjugate graph [34] and inverse graph [35]. Inverse graph of a group is first introduced by Alfuraidan and Zakariya [35] in 2017. Suppose that Γ is a finite group and S is a set of non-self-invertible elements in Γ, namely $S = \{u \in \Gamma: u \neq u^{-1}\}$. The inverse graph of Γ is denoted by $G_S(\Gamma)$ and is defined as a graph whose set of vertex is the set Γ and two different elements v and w of Γ will be joined by an edge in $G_S(\Gamma)$ if and only if either vw or wv are elements of S.

Several studies on the spectra of graph obtained from a finite group have been conducted and published. For dihedral group, the spectra of non-commuting and commuting graphs [36], conjugate graph and its complement [37,38] and subgroup graph and its complement [39-41] have been reported. Nevertheless, study on the spectra of inverse graph of the symmetry group and the addition group of integers modulo n has not been found.

Edwina and Sugeng [42] introduced the notion of anti-adjacency matrix. Suppose that graph G is simple (without loops or multiple edges) with order p and $A(G) = [a_{ij}]$ $(1 \le i,j \le p)$ is its adjacency matrix. The anti-adjacency matrix of graph G is matrix $B(G) = [b_{ij}]$ $(1 \le i,j \le p)$ where $b_{ij} = 1$ if $a_{ij} = 0$ and $b_{ij} = 0$ if $a_{ij} = 1$. So, anti-adjacency matrix $B(G)$ can be expressed as $B(G) = J - A(G)$ where J is $p \times p$ matrix whose all entries are one [43]. In other words, the anti-adjacency matrix $B(G)$ is the opposite of the matrix $A(G)$ [44]. Until now, no one has examined the anti-adjacency spectrum of graphs, especially of graphs associated with a group.

This study examines the spectrum of anti-adjacency and Laplacian matrices of the inverse graph of a group. This study focuses on the addition group of integers modulo n where n is a positive integer.

2. Literature Review

Suppose G is a simple and finite graph with order $p = |\boldsymbol{V(G)}|$ and size $q = |\boldsymbol{E(G)}|$. Suppose $V(G) = \{v_i : 1 \le i \le p\}$. The degree $\deg(v_i)$ of a vertex v_i in G is defined as the number of vertex v_j $(j \ne i)$ in G such that v_iv_j is an element of $E(G)$ [45]. The matrix $A(G) = [a_{ij}]$ $(1 \le i,j \le p)$ where $a_{ij} = 1$ if v_iv_j is an element of $E(G)$ and $a_{ij} = 0$ if v_iv_j is not element of $E(G)$ is called the adjacency matrix of a graph G [46]. The matrix $D(G) = [d_{ij}]$ $(1 \le i,j \le p)$ where $d_{ij} = \deg(v_i)$ if $i = j$ and $d_{ij} = 0$ if $i \ne j$ is called the degree matrix of graph G [47]. The matrix $L(G) = D(G) - A(G)$ is called the Laplacian matrix of graph G [2,4]. The characteristic polynomial of $A(G)$ is a polynomial $\sigma(\lambda) = \det(A(G) - \lambda I)$ where I is $p \times p$ identity matrix [48]. The roots of $\sigma(\lambda) = 0$ are eigenvalues of $A(G)$ [49]. Suppose $\lambda_1 > \lambda_2 > \ldots > \lambda_k$ $(k \le p)$ are the distinct eigenvalues of $A(G)$ and $m(\lambda_i)$ is the algebraic multiplicity associated with λ_i $(1 \le i \le k)$. The adjacency spectrum $\boldsymbol{spec_A(G)}$ of a graph G is a $2 \times k$ matrix that contains the distinct eigenvalues of $A(G)$ in the first row and their corresponding multiplicities in the second row [50]. The adjacency spectrum of G can be written as

$$spec_A(G) = \begin{bmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_k \\ m(\lambda_1) & m(\lambda_2) & \cdots & m(\lambda_k) \end{bmatrix} \quad (1)$$

In a similar way, the Laplacian spectrum $spec_L(G)$ of a graph G is obtained from matrix $L(G)$ [51] while the anti-adjacency spectrum $spec_B(G)$ of a graph G is obtained from matrix $B(G)$. If all of the eigenvalues of $L(G)$ are integer, then graph G is called integral [40].

3. Results

Suppose $(Z_n, +)$ is the addition group of integers modulo n where n is positive integer. It is well known that $Z_n = \{\overline{0}, \overline{1}, \overline{2}, \ldots, \overline{n-1}\}$ and $\overline{0}$ is identity element of Z_n. If n is odd, then $\overline{0}$ is the only self-invertible element of Z_n. Hence, the set of non-self-invertible elements S of Z_n is $S = \{\overline{1}, \overline{2}, \ldots, \overline{n-1}\} = Z_n \backslash \{\overline{0}\}$. If n is even, then $\overline{0}$ and $\overline{\frac{n}{2}}$ are self-invertible elements of Z_n. Therefore, the set of non-self-invertible elements S of Z_n is $S = \{\overline{1}, \overline{2}, \ldots, \overline{\frac{n}{2}-1}, \overline{\frac{n}{2}+1}, \ldots, \overline{n-2}, \overline{n-1}\} = Z_n \backslash \{\overline{0}, \overline{\frac{n}{2}}\}$. According to the definition of inverse graph of Z_n, it is straightforward that $\deg(\overline{0}) = |S| - 1 = n - 1$ if n is odd and $\deg(\overline{0}) = |S| - 1 = \deg(\overline{\frac{n}{2}})$ if n is even. Furthermore, $\deg(v) = |S| - 2 = n - 2$ for $v \in Z_n \backslash \{\overline{0}\}$ if n is odd.

The results of the present study on the inverse graph of the addition group Z_n are presented as the following. First, the results will be presented with proof.

Theorem 3.1

The characteristic polynomial of anti-adjacency matrix $B(G_s(Z_n))$ is

$$\sigma(\lambda) = (\lambda-2)^{\frac{n-1}{2}}(\lambda-1)(\lambda)^{\frac{n-1}{2}} \quad (2)$$

if n is odd.

Proof

Because n is odd, then the set of all non-self-invertible elements of group Z_n is $S = \{\overline{1}, \overline{2}, \ldots, \overline{k}, \ldots, \overline{n-k}, \ldots \overline{n-2}, \overline{n-1}\}$. In the inverse graph $G_s(Z_n)$, the vertex $\overline{k}$ is not adjacent to vertex $\overline{n-k}$ $(1 \le k < n)$ and is adjacent to all other vertices. Therefore, the adjacency matrix of $G_s(Z_n)$ is $A(G_S(Z_n)) = [a_{ij}]$ where

$$a_{ij} = \begin{cases} 1, \text{ if } i=j \text{ or } i=t \text{ and } j=n-(t-2)\ (1< t \le n) \\ 0, \text{ otherwise} \end{cases}$$

and the anti-adjacency matrix of $G_s(Z_n)$ is $B(G_S(Z_n)) = [b_{ij}]$

$$b_{ij} = \begin{cases} 0, \text{ if } i=j \text{ or } i=t \text{ and } j=n-(t-2)\ (1< t \le n) \\ 1, \text{ otherwise} \end{cases}$$

The matrix $B(G_S(Z_n))$ can be presented as

$$\mathrm{B}(\mathrm{G_s}(\mathrm{Z_n})) = \begin{array}{c} \\ \overline{0} \\ \overline{1} \\ \overline{2} \\ \vdots \\ \overline{k} \\ \vdots \\ \overline{n-k} \\ \vdots \\ \overline{n-2} \\ \overline{n-1} \end{array} \begin{array}{c} \begin{array}{cccccccccc} \overline{0} & \overline{1} & \overline{2} & \ldots & \overline{k} & \ldots & \overline{n\text{-}k} & \ldots & \overline{n\text{-}2} & \overline{n\text{-}1} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & 0 \\ 0 & 1 & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & 1 \\ 0 & 0 & 1 & \ldots & 0 & \ldots & 0 & \ldots & 1 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 1 & \ldots & 1 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 1 & \ldots & 1 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 1 & \ldots & 0 & \ldots & 0 & \ldots & 1 & 0 \\ 0 & 1 & 0 & \ldots & 0 & \ldots & 0 & \ldots & 0 & 1 \end{bmatrix} \end{array} \quad (3)$$

By using Gaussian elimination on $B(G_s(\mathrm{Z_n})) - \lambda I$, an upper triangular matrix U will be obtained. Therefore, $\sigma(\lambda) = \det(B(\mathrm{G_s}(\mathrm{Z}_n)) - \lambda I)$ can be obtained by multiplying entries along the main diagonal of U. Finally, by simplifying the results of multiplication, it will be found that $\sigma(\lambda) = (\lambda-2)^{\frac{n-1}{2}}(\lambda-1)(\lambda)^{\frac{n-1}{2}}$.

Corollary 3.1

The anti-adjacency spectrum of $G_S(Z_n)$ is

$$spec_B(\mathrm{G_s}(\mathrm{Z_n})) = \begin{bmatrix} 2 & 1 & 0 \\ \frac{n-1}{2} & 1 & \frac{n-1}{2} \end{bmatrix} \quad (4)$$

if n is odd.

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Proof

Based on Theorem 3.1, the different eigenvalues of $B(G_S(Z_n))$ are $\lambda_1 = 2,\ \lambda_2 = 1$ and $\lambda_3 = 0$ and their multiplicities are $m(\lambda_1) = \frac{n-1}{2},\ m(\lambda_2) = 1$ and $m(\lambda_3) = \frac{n-1}{2}$, respectively. By the definition of the spectrum of a graph, it is obvious that

$$spec_B(\mathrm{G_s(Z_n)}) = \begin{bmatrix} 2 & 1 & 0 \\ \frac{n-1}{2} & 1 & \frac{n-1}{2} \end{bmatrix} \tag{5}$$

Theorem 3.2

The characteristic polynomial of $L(G_s(Z_n))$ is

$$\sigma(\lambda) = (\lambda - n)^{\frac{n-1}{2}}(\lambda - n + 2)^{\frac{n-1}{2}}(\lambda) \tag{6}$$

if n is odd.

Proof

Previously explained that $\deg(\bar{0}) = |S| - 1 = n - 1$ and $\deg(v) = |S| - 2 = n - 2$ for $v \in Z_n\backslash\{\bar{0}\}$ if n is odd. Hence, the degree matrix of $G_s(Z_n)$ is $D(G_S(Z_n)) = [d_{ij}]$ where

$$d_{ij} = \begin{cases} n-1, \text{ if } i=j=1 \\ n-2, \text{ if } i=j\neq 1 \\ 0, i\neq j \end{cases}$$

Thus, the matrix $L(\Gamma_s(\mathrm{Z_n})) = D(\Gamma_s(\mathrm{Z_n}))$ - $A(\Gamma_s(\mathrm{Z_n}))$ is

$$\mathrm{L}(\Gamma_s(\mathrm{Z_n})) = \begin{array}{c} \\ \bar{0} \\ \bar{1} \\ \bar{2} \\ \vdots \\ \bar{k} \\ \vdots \\ \overline{n-k} \\ \vdots \\ \overline{n-2} \\ \overline{n-1} \end{array} \begin{array}{c} \begin{array}{cccccccccc} \bar{0} & \bar{1} & \bar{2} & \dots & \bar{k} & \dots & \overline{n-k} & \dots & \overline{n-2} & \overline{n-1} \end{array} \\ \begin{bmatrix} n-1 & -1 & -1 & \dots & -1 & \dots & -1 & \dots & -1 & -1 \\ -1 & n-2 & -1 & \dots & -1 & \dots & -1 & \dots & -1 & 0 \\ -1 & -1 & n-2 & \dots & -1 & \dots & -1 & \dots & 0 & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ -1 & -1 & -1 & \dots & n-2 & \dots & 0 & \dots & -1 & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ -1 & -1 & -1 & \dots & 0 & \dots & n-2 & \dots & -1 & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ -1 & -1 & 0 & \dots & -1 & \dots & -1 & \dots & n-2 & -1 \\ -1 & 0 & -1 & \dots & -1 & \dots & -1 & \dots & -1 & n-2 \end{bmatrix} \end{array} \tag{7}$$

Through Gaussian elimination on $L(G_s(\mathrm{Z}_n)) - \lambda I$ and some computation will be obtained $\sigma(\lambda) = \det(L(G_s(\mathrm{Z_n})) - \lambda I) = (\lambda - n)^{\frac{n-1}{2}}(\lambda - n + 2)^{\frac{n-1}{2}}(\lambda)$.

Corollary 3.2

The Laplacian spectrum of $G_S(Z_n)$ is

$$spec_L(G_s(Z_n)) = \begin{bmatrix} n & n-2 & 0 \\ \frac{n-1}{2} & \frac{n-1}{2} & 1 \end{bmatrix} \tag{8}$$

if n is odd.

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Proof

It is obvious from Theorem 3.2,

If n is even, then $n \equiv 0 \pmod 4$ or $n \equiv 2 \pmod 4$. Henceforth, the main results of this study are presented without proof.

Theorem 3.3

The characteristic polynomial of $B(G_s(Z_n))$ is

$$\sigma(\lambda) = (\lambda - 3)^{\frac{n-4}{4}}(\lambda - 2)^2(\lambda - 1)^{\frac{n-4}{2}}(\lambda)^2(\lambda + 1)^{\frac{n-4}{4}} \quad (9)$$

if $n \geq 8$ and $n \equiv 0 \pmod 4$.

Corollary 3.3

The anti-adjacency spectrum of $G_S(Z_n)$ is

$$Spec_B\big(\mathrm{G_s(Z_n)}\big) = \begin{bmatrix} 3 & 2 & 1 & 0 & -1 \\ \frac{n-4}{4} & 2 & \frac{n-4}{2} & 2 & \frac{n-4}{4} \end{bmatrix} \quad (10)$$

if $n \geq 8$ and $n \equiv 0 \pmod 4$.

Theorem 3.4

The characteristic polynomial of $L(G_s(Z_n))$ is

$$\sigma(\lambda) = (\lambda - n)^{\frac{n}{4}}(\lambda - n + 2)^{\frac{n}{2}}(\lambda - n + 4)^{\frac{n-4}{4}}(\lambda) \quad (11)$$

if $n \geq 8$ and $n \equiv 0 \pmod 4$.

Corollary 3.4

The Laplacian spectrum of $G_S(Z_n)$ is

$$spec_L\big(\mathrm{G_s(Z_n)}\big) = \begin{bmatrix} n & n-2 & n-4 & 0 \\ \frac{n}{4} & \frac{n}{2} & \frac{n-4}{4} & 1 \end{bmatrix} \quad (12)$$

if $n \geq 8$ and $n \equiv 0 \pmod 4$.

Theorem 3.5

The characteristic polynomial of $B(G_s(Z_n))$ is

$$\sigma(\lambda) = (\lambda - 3)^{\frac{n-2}{4}}(\lambda - 2)(\lambda - 1)^{\frac{n-2}{2}}(\lambda)(\lambda + 1)^{\frac{n-2}{4}} \quad (13)$$

if $n \equiv 2 \pmod 4$.

Corollary 3.5

The anti-adjacency spectrum of $G_S(Z_n)$ is

$$spec_B\big(\mathrm{G_s(Z_n)}\big) = \begin{bmatrix} 3 & 2 & 1 & 0 & -1 \\ \frac{n-2}{4} & 1 & \frac{n-2}{2} & 1 & \frac{n-2}{4} \end{bmatrix} \tag{14}$$

if $n \equiv 2 \pmod 4$.

Theorem 3.6

The characteristic polynomial of $L(G_s(Z_n))$ is

$$\sigma(\lambda) = (\lambda - n)^{\frac{n-2}{4}}(\lambda - n + 2)^{\frac{n}{2}}(\lambda - n + 4)^{\frac{n-2}{4}}(\lambda) \tag{15}$$

if $n \equiv 2 \pmod 4$.

Corollary 3.6

The Laplacian spectrum of $G_S(Z_n)$ is

$$spec_L\big(\mathrm{G_s(Z_n)}\big) = \begin{bmatrix} n & n-2 & n-4 & 0 \\ \frac{n-2}{4} & \frac{n}{2} & \frac{n-2}{4} & 1 \end{bmatrix} \tag{16}$$

if $n \equiv 2 \pmod 4$.

4. Conclusion

According to the results of this study, it can be seen that all eigenvalues of anti-adjacency and Laplacian matrices of the inverse graph of addition group of integers modulo *n* are integer. So, the inverse graph of this group is integral. The next research can be done to examined the other spectrum of inverse graph of this group or other groups.

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