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Survey on topological indices and graphs associated with a commutative ring

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Survey on topological indices and graphs associated with a commutative ring

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Abstract. The researches on topological indices are initially related to graphs obtained from biological activities or chemical structures and reactivity. Recently, the research on this topic has evolved on graphs in general and even on graphs obtained from algebraic structures, such as groups, rings or modules. This paper will present various topological index concepts, various graph concepts obtained from a commutative ring and some previous studies that are relevant to those two concepts. Based on the various concepts presented, research topics related to topological indices of a graph associated with a commutative ring can be found and carried out.

1. Introduction

Topological index is still a topic of interest to researchers. When a researcher conducts research related to a concept of topological index, new topological index concepts continue to be developed and introduced by other researchers. Thus, the research on this topic is endless. Although topological indices were originally developed for graphs obtained from representations of chemical structures, researches of topological indices are not limited to them. Some topological index studies on graphs in general have also been carried out, for example on connected graphs [1], bridge graphs [2], thorn graphs [3], trees [4–6], unicyclic graphs [7], bipartite graphs [8], composite graphs [9], windmill graph [10], Sierpinski graph [11] and graphs obtained from certain graph operations [12].

On the other hand, new types of graphs were also developed and introduced. The researchers began to introduce the concept of graphs related to algebraic structures. The graphs were developed from the group, for example commuting [13] and non-commuting graph [14], inverse graphs [15], identity graphs [16], conjugate graphs [17] and subgroup graphs [18]. Several studies on topological indices of graphs obtained from a group have been reported, for example studies on non-commuting graphs of a finite group [19–24], subgroup graph of dihedral group [25,26] and identity graph of cyclic group [27]. However, researches on topological indices of graphs associated with a ring is still rarely done.

This paper presents the concepts of topological indices and the concepts of graph associated with a ring, especially a commutative ring. Previous researches relevant to these concepts are also presented. This paper is expected to help researchers who are interested in conducting research related to the topological indices of graphs obtained from a commutative ring.

2. Topological Indices of a Graph

Throughout this paper, graph $G = (V(G), E(G))$ is simple and finite. Let $m = |V(G)|$ and $n = |E(G)|$ are the order and the size of G, respectively. Let $N(x)$ denoted the open neighborhood of a vertex x in G. Then, the degree of x is $\deg(x) = |N(x)|$ and the closed neighborhood of x is $N[x] = N(x) \cup \{x\}$. The number of distinct edges that incident to any vertex in $N[x]$ is defined as the ve-degree of x and is denoted by $\deg_{ve}(x)$. The distance between any two vertices x and y in G is denoted by $d(x, y)$. The eccentricity of a vertex x in G is denoted by $e(x)$ and is defined as the largest $d(x, y)$ for any y in G. For any vertex x of G, the total distance of x is defined as $D(x) = \sum_{y \in V(G)} d(x, y)$. The diameter of G is denoted by $\text{diam}(G)$ and is defined as the greatest distance between any two vertices in G.

The topological index of a graph is a number that invariant under graph automorphism [28]. Topological index also called as molecular structure descriptor [29] or graph-theoretical descriptor [30–32]. The degree-based, distance-based and eccentricity-based topological indices are three major classifications. Following are some definitions of topological indices. However, it is possible that there are still other definitions that are not covered in this paper.

The first and second Zagreb indices are defined as [33,34]

$$M_1(G) = \sum_{x \in V(G)} \deg(x)^2 = \sum_{xy \in E(G)} [\deg(x) + \deg(y)]$$

and

$$M_2(G) = \sum_{xy \in E(G)} \deg(x) \deg(y).$$

In a more general form, the first general Zagreb index is defined as [35]

$$M_1^\alpha(G) = \sum_{x \in V(G)} \deg(x)^\alpha = \sum_{xy \in E(G)} [\deg(x)^{\alpha-1} + \deg(y)^{\alpha-1}]$$

for any $\alpha \in \mathbb{R}$. For $\alpha = 3$, it is called as the forgotten topological index or F-index and is written as [36,37]

$$F(G) = \sum_{x \in V(G)} \deg(x)^3 = \sum_{xy \in E(G)} [\deg(x)^2 + \deg(y)^2].$$

The first and second Zagreb co-indices are defined as [4,35]

$$\overline{M_1}(G) = \sum_{xy \notin E(G)} [\deg(x) + \deg(y)] \text{ and } \overline{M_2}(G) = \sum_{xy \notin E(G)} \deg(x) \deg(y)$$

The first and second ve-degree Zagreb indices are defined as [38]

$$M_{ve_1}(G) = \sum_{xy \in E(G)} [\deg_{ve}(x) + \deg_{ve}(y)] \text{ and } M_{ve_2}(G) = \sum_{xy \in E(G)} \deg_{ve}(x) \deg_{ve}(y)$$

The F-ve-degree index is defined as [38]

$$F_{ve}(G) = \sum_{x \in V(G)} \deg_{ve}(x)^3 = \sum_{xy \in E(G)} [\deg_{ve}(x)^2 + \deg_{ve}(y)^2].$$

The reduced second Zagreb index is defined as [25]

$$RM_2(G) = \sum_{xy \in E(G)} [\deg(x) - 1][\deg(y) - 1]$$

and the Narumi-Katayama index is defined as [39]

$$NK(G) = \prod_{x \in V(G)} \deg(x).$$

In addition, the first multiplicative Zagreb index is defined as [25]

$$\Pi_1(G) = \prod_{x \in V(G)} \deg(x)^2 = \prod_{xy \in E(G)} (\deg(x) + \deg(y))$$

and it implies that $\Pi_1(G) = NK(G)^2$. The second multiplicative Zagreb index is defined as [25]

$$\Pi_2(G) = \prod_{xy \in E(G)} \deg(x) \deg(y).$$

There are two definitions of Randic index [40], namely

$$R(G) = \sum_{xy \in E(G)} \frac{1}{\deg(x) \deg(y)} = \sum_{xy \in E(G)} (\deg(x) \deg(y))^{-1}$$

and

$$R(G) = \sum_{xy \in E(G)} \frac{1}{\sqrt{\deg(x) \deg(y)}} = \sum_{xy \in E(G)} (\deg(x) \deg(y))^{-\frac{1}{2}}.$$

The Randic index is also well known as branching index or connectivity index or product connectivity index. From Randic index, some definitions are introduced. The reciprocal Randic index, reduced reciprocal Randic Index and the general Randic index are defined as the following.

$$RR(G) = \sum_{xy \in E(G)} \sqrt{\deg(x)\deg(y)},$$

$$RRR(G) = \sum_{xy \in E(G)} \sqrt{[\deg(x) - 1][\deg(y) - 1]}, \text{ and}$$

$$R_\alpha(G) = \sum_{xy \in E(G)} (\deg(x)\deg(y))^\alpha$$

for an arbitrary $\alpha \in \mathbb{R}$. Therefore, $R_1(G) = M_2(G)$. The general Randic co-index is defined as

$$\overline{R_\alpha}(G) = \sum_{xy \notin E(G)} (\deg(x)\deg(y))^\alpha$$

for an arbitrary $\alpha \in \mathbb{R}$. It is also called the general product connectivity index.

The ABC-index or Atom-Bond-Connectivity index is defined as [41]

$$ABC(G) = \sum_{xy \in E(G)} \sqrt{\frac{\deg(x) + \deg(y) - 2}{\deg(x)\deg(y)}}$$

The harmonic index is defined as [42–45]

$$H(G) = \sum_{xy \in E(G)} \frac{2}{\deg(x) + \deg(y)}$$

The augmented Zagreb index is defined as [46]

$$AZI(G) = \sum_{xy \in E(G)} \left(\frac{\deg(x)\deg(y)}{\deg(x) + \deg(y) - 2} \right)^3$$

The first geometric-arithmetic index or geometric-arithmetic index is defined as [47]

$$GA(G) = \sum_{xy \in E(G)} \frac{\sqrt{\deg(x)\deg(y)}}{\frac{(\deg(x) + \deg(y))}{2}} = \sum_{xy \in E(G)} \frac{2\sqrt{\deg(x)\deg(y)}}{\deg(x) + \deg(y)}$$

The sum-connectivity and general sum-connectivity indices are defined as [48,49]

$$SCI(G) = \sum_{xy \in E(G)} \frac{1}{\sqrt{\deg(x) + \deg(y)}} = \sum_{xy \in E(G)} (\deg(x) + \deg(y))^{-\frac{1}{2}}$$

and

$$SCI_\alpha(G) = \sum_{xy \in E(G)} (\deg(x) + \deg(y))^\alpha$$

for any $\alpha \in \mathbb{R}$. The general sum-connectivity co-index is defined as

$$\overline{SCI_\alpha}(G) = \sum_{xy \notin E(G)} (\deg(x) + \deg(y))^\alpha$$

where α is real number [35].

All topological indices described above are the degree-based topological indices of a graph. The following is the distance-based topological indices of a graph. The first is the Wiener index which is defined as [50]

$$W(G) = \sum_{\{x,y\} \subseteq V(G)} d(x,y).$$

The Wiener polarity index is defined as [51]

$$W_P(G) = d(G, 3)$$

where $d(G, 3)$ denote the number of vertex pairs in G that has the distance 3. The terminal Wiener index is defined as

$$TW(G) = \sum_{\{x,y\} \subseteq V_1(G)} d(x,y)$$

where $V_1(G)$ is the set of end vertices in G [52]. The hyper Wiener index is defined as [19]

$$WW(G) = \frac{1}{2}\sum_{\{x,y\}\subseteq V(G)}[d(x,y) + d(x,y)^2]$$

and the reciprocal complementary Wiener index is defined as [53]

$$RCW(G) = \sum_{\{x,y\}\subseteq V(G)}\frac{1}{\mathrm{diam}(G)-1+d(x,y)}.$$

The old Harary index is defined as [54]

$$H_{old}(G) = \sum_{\{x,y\}\subseteq V(G)}\frac{1}{d(x,y)^2}$$

while the Harary index is defined as [55,56]

$$H(G) = \sum_{\{x,y\}\subseteq V(G)}\frac{1}{d(x,y)}.$$

Next, the eccentricity-based topological indices of a graph G are presented. The first, total eccentricity of G is defined as [57,58]

$$\xi(G) = \sum_{x\in V(G)}e(x).$$

The first and second Zagreb eccentricity indices are defined as [25]

$$E_1(G) = \sum_{x\in V(G)}(e(x))^2 \text{ and } E_2(G) = \sum_{xy\in E(G)}e(x)e(y)$$

The eccentric connectivity is

$$\xi^c(G) = \sum_{x\in V(G)}e(x)\deg(x)$$

and connective eccentricity index is

$$\text{and } C^\xi(G) = \sum_{x\in V(G)}\frac{\deg(x)}{e(x)} \text{ [25]}$$

The eccentric distance sum index is defined as [25,26]

$$\xi^d(G) = \sum_{x\in V(G)}e(x)D(x) \text{ or } \xi^d(G) = \sum_{\{x,y\}\subseteq V(G)}[e(x) + e(y)]d(x,y).$$

while the adjacent eccentric distance sum index is defined as [25,26]

$$\xi^{sv}(G) = \sum_{x\in V(G)}\frac{e(x)D(x)}{\deg(x)}.$$

The Schultz index or degree distance index is defined as [59]

$$DD(G) = \sum_{x\neq y}(\deg(x) + \deg(y))d(x,y)$$

and Gutman index is defined as [60,61]

$$\mathrm{Gut}(G) = \sum_{x\neq y}\deg(x)\deg(y)d(x,y).$$

The additively weighted Harary index or reciprocal degree distance index is defined as [62]

$$H_A(G) = \sum_{x\neq y}\frac{(\deg(x)+\deg(y))}{d(x,y)}$$

while multiplicatively weighted Harary index is defined as [63–65]

$$H_M(G) = \sum_{x\neq y}\frac{(\deg(x)\deg(y))}{d(x,y)}.$$

After knowing the various concepts of the topological index, then the next is knowing the concepts of graphs related to a commutative ring. With this knowledge, it is possible for a researcher to conduct research on topological indices of a graph associated with a commutative ring.

3. Graphs Associated with a Commutative Ring

Let R be a commutative ring with unity $1 \neq 0$. Let $Z(R)$ denotes the set of zero divisors, $Z(R)\backslash\{0\}$ denotes the set of non-zero zero divisors and $U(R)$ denotes the set of units in R. The annihilator of an element r is the set $Ann(r) = \{s \in R \mid rs = 0\}$. A non-zero divisor element in R is called regular element. An element r is called a nilpotent in R if $r^n = 0$ for some positive integer n.

The following are definitions of some graphs obtained from R.

The zero divisor graph of R is the graph with its vertex set is the set $Z(R) *$ and two different vertices r and s are joined by an edge if and only if $rs = 0$[66].

The total graph of R is the graph with its vertex set is R and two different vertices r and s are joined by an edge if and only if $r + s \in Z(R)$ [67].

The total zero divisor graph of R is the graph with its vertex set is the set $Z(R)^*$, and two different vertices r and s are joined by an edge if and only if $rs = 0$ and $r + s \in Z(R)$ [68].

The co-zero divisor graph of R is the graph with its vertex set is the set of all non-zero and non-unit elements in R and two different vertices r and s are adjacent if and only if $r \notin Rs$ and $s \notin Rr$ [69].

The unit graph of R is the graph with its vertex set is R and two different vertices r and s are adjacent if and only if $r + s \in U(R)$ [67].

The identity graph of R is the graph with its vertex set is $U(R)$ and two different vertices r and s are joined by an edge if and only if $rs = 1$ [27].

Let S be a subset of R that is closed to multiplication operations. The generalized total graph of R is the graph with its vertex set is R and two different vertices r and s are joined by an edge if and only if $r + s \in S$. If $S = Z(R)$ then this is the total graph of R. If $S = U(R)$ then this is the unit graph of R [70].

The nilradical graph of R is the graph with its vertex set is the set of non-zero nilpotents of R and two distinct vertices r and s are joined by an edge if and only if $rs = 0$. The non-nilradical graph of R is the graph with its vertex set is the set of non-nilpotent zero-divisors of R and two different vertices r and s are joined by an edge if and only if $rs = 0$ [71].

The annihilator graph of R is the graph with its vertex set is $Z(R) *$ and two distinct vertices r and s are joined by an edge if and only if $Ann(r) \cup Ann(s) \neq Ann(rs)$[72].

The maximal graph of R is the graph with its vertex set is R and two distinct vertices r and s are adjacent if and only if r and s are elements of a maximal ideal M of R[73]. The co-maximal graph of R is the graph with the vertex set R and two distinct vertices r and s are joined by an edge if and only if $rR + sR = R$[74].

The containment graph of R is the graph with its vertex set is the set of the ideals in R and two distinct vertices I and J are joined by an edge if and only if $I \subset J$. The intersection graph of ideals of R is the graph with its vertex set is the set of the non-trivial proper ideals and two distinct vertices I and J are joined by an edge if and only if $I \neq J$ and $I \cap J \neq \{0\}$ [75].

4. Topological Indices of a Graph Associated with a Commutative Ring

The research on topological indices of a graph obtained from a commutative ring is still infrequent. One study that has examined this topic is done by Abdussakir et al [27]. They determined the eccentric connectivity index of the identity graph of a commutative ring. As an example, several topological indices of the identity graph of ring of integer modulo p will be determined in this paper, where p is a prime number.

For any prime number p, let $(Z_p, +, \cdot)$ is the ring of integer modulo p. Then Z_p is a field and all the non-zero elements of Z_p are units. Based on the definition of identity graph, $I(Z_2)$ is a trivial graph and $I(Z_3)$ is a path of order 2. For $p > 3$, $I(Z_p)$ as the following [27].
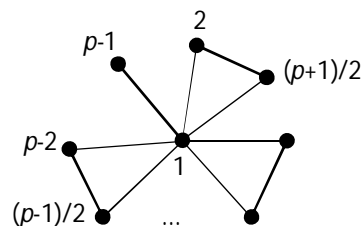

Figure 1. Identity graph of Z_p

Then, $\deg(1) = p - 2$, $\deg(p - 1) = 1$ and $\deg(v) = 2$ for $v \neq 1$ or $v \neq p - 1$. It will be easily observed that $e(1) = 1$ and $e(v) = 2$ for $v \neq 1$. It also obtained that $D(1) = p - 2$, $D(p - 1) = 2p - 5$ and $D(v) = 2p - 6$ for $v \neq 1$ or $v \neq p - 1$. According to these facts, the following results are obtained.

Theorem 4.1

    (a) $M_1(I(Z_2)) = 0$ and $M_1(I(Z_3)) = 2$

    (b) $M_1(I(Z_p)) = p^2 - 5$, for $p > 3$

    (c) $M_2(I(Z_2)) = 0$ and $M_2(I(Z_3)) = 1$

    (d) $M_2(I(Z_p)) = 3p - 8$, for $p > 3$

    (e) $F(I(Z_2)) = 0$ and $F(I(Z_3)) = 2$

    (f) $F(I(Z_p)) = p^3 - 6p^2 + 20p - 31$, for $p > 3$

    (g) $NK(I(Z_2)) = 0$ and $M_1(I(Z_3)) = 1$

    (h) $NK(I(Z_p)) = 2^{p-3}(p - 2)$, for $p > 3$

    (i) $\Pi_1(I(Z_2)) = 0$ and $\Pi_1(I(Z_3)) = 2$

    (j) $\Pi_1(I(Z_p)) = p^2 - 5$, for $p > 3$

    (k) $\xi(I(Z_2)) = 0$ and $\xi(I(Z_3)) = 2$

    (l) $\xi(I(Z_p)) = 2p - 3$, for $p > 3$

    (m) $\xi^c(I(Z_2)) = 0$ and $\xi^c(I(Z_3)) = 2$

    (n) $\xi^c(I(Z_p)) = 2 = 5p - 12$, for $p > 3$

Proof. For $p = 2$ and $p = 3$, the proofs are obvious. For $p > 3$, then

    (b) $M_1(I(Z_p)) = \sum_{u \in V(I(Z_p))} \deg(u)^2 = \deg(1)^2 + \deg(p - 1)^2 + \sum_{\substack{u \neq 1 \\ u \neq p-1}} \deg(u)^2 = (p - 2)^2 + 1 + (p - 3)2^2 = p^2 - 5$.

    (d) $M_2(G) = \sum_{uv \in E(G)} \deg(u)\deg(v) = \sum_{\substack{1v \in E(G) \\ v \neq 1}} \deg(1)\deg(v) + \sum_{\substack{uu^{-1} \in E(G) \\ u \neq 1, u \neq p-1}} \deg(u)\deg(v) = \left[1 \cdot 1 + \frac{p-3}{2}(1 \cdot 2) \right] + \left[\frac{p-3}{2}(2 \cdot 2) \right] = 3p - 8$.

    (f) $F(G) = \sum_{u \in V(G)} \deg(u)^3 = \deg(1)^3 + \deg(p - 1)^3 + \sum_{\substack{u \neq 1 \\ u \neq p-1}} \deg(u)^3 = (p - 2)^3 + 1 + (p - 3)2^3 = (p - 2)^3 + 8p - 23 = p^3 - 6p^2 + 20p - 31$.

    (h) $NK(G) = \prod_{v \in V(G)} \deg(v) = \deg(1)\deg(p - 1) \prod_{\substack{u \neq 1 \\ u \neq p-1}} \deg(u) = (p - 2) \cdot 1 \cdot 2^{p-3} = 2^{p-3}(p - 2)$.

    (j) $\Pi_1(G) = \prod_{v \in V(G)} \deg(v)^2 = \deg(1)^2 \deg(p - 1)^2 \prod_{\substack{u \neq 1 \\ u \neq p-1}} \deg(u)^2 = (p - 2)^2 \cdot 1 \cdot 2^{2(p-3)} = 2^{2(p-3)}(p - 2)^2$.

The proof of (l) and (n) can be seen in [27].

5. Conclusion

This paper presented various definitions of topological indices and graphs obtained from a commutative ring. Of course, there are still topological indices and graphs obtained from the ring that have not been presented in this paper. Nevertheless, the presence of this paper can help researchers to conduct research on the topic of topological indices of graphs related to a commutative ring.

References

[1]   Bielak H and Broniszewska K 2017 Eccentric distance sum index for some classes of connected graphs *Ann. Univ. Mariae Curie-Sklodowska Lublin-Polonia* **LXXI** 25–32

[2]   Mogharrab M 2010 Eccentric connectivity index of bridge graphs *Optoelectron. Adv. Mater. Commun.* **4** 1866–7

[3]   De N 2012 On eccentric connectivity index and polynomial of thorn graph *Appl. Math.* **3** 931

[4]   Rasi R, Sheikholeslami S M and Behmaram A 2017 An upper bound on the first Zagreb index in trees *Iran. J. Math. Chemstry* **8** 71–82

[5]   Geng X, Li S and Zhang M 2013 Extremal values on the eccentric distance sum of trees ☆ *Discret. Appl. Math.* **161** 2427–39

[6]   Yu G, Feng L and Ilic A 2011 On the eccentric distance sum of trees and unicyclic graphs ☆ *J. Math. Anal. Appl.* **375** 99–107

[7]   Nacaroglu Y and Maden A D 2018 On the eccentric connectivity index of unicyclic graphs *Iran. J. Math. Chem.* **9** 47–56

[8]   Li S C, Wu Y Y and Sun L L 2015 On the minimum eccentric distance sum of bipartite graphs with some given parameters *J. Math. Anal. Appl.* **430** 1149–62

[9]   Došlic T and Saheli M 2014 Eccentric connectivity index of composite graphs *Util. Math* **95** 3–22

[10] Kanna M R R, Kumar P and Nandappa D S 2018 Computation of topological indices of windmill grahp *Int. J. Pure Appl. Math.* **119** 89–98

[11] Zhang X 2017 Some degree-based topological indices of base-3 sierpiński graphs *Sci. J. Chem.* **5** 36

[12] Eskender B and Vumar E 2013 Eccentric connectivity index and eccentric distance sum of some graph operations *Trancaction Comb.* **2** 103–11

[13] Vahidi J and Talebi A A 2010 The commuting graphs on groups D2n and Qn *J. Math. Comput. Sci.* **1** 123–7

[14] Raza Z and Faizi S 2013 Non-commuting graph of a finitely presented group *Sci. Int.* **25** 883–5

[15] Alfuraidan M R and Zakariya Y F 2017 Inverse graphs associated with finite groups *Electron. J. Graph Theory Appl.* **5** 142–54

[16] Kandasamy W B V and Smarandache F 2009 *Groups as graphs* (Judetul Olt, Romania: Editura CuArt)

[17] Erfanian A and Tolue B 2012 Conjugate graphs of finite groups *Discret. Math. Algorithms Appl.* **04** 1–8

[18] Anderson D F, Fasteen J and Lagrange J D 2012 The subgroup graph of a group *Arab J. Math.* **1** 17–27

[19] Mirzargar M and Ashrafi A R 2012 Some distance-based topological indices of a non-commuting graph *Hacettepe J. Math. Stat.* **41** 515–26

[20] Sarmin N H, Alimon N I and Erfanian A 2019 Topological Indices of the Non-commuting Graph for Generalised Quaternion Group *Bull. Malaysian Math. Sci. Soc.* 1–7

[21] Alimon N I, Sarmin N H and Erfanian A The topological indices of non-commuting graph for quasidihedral group

[22] Alimon N I, Sarmin N H and Erfanian A 2018 Topological indices of non-commuting graph of dihedral groups *Malaysian J. Fundam. Appl. Sci.* **14** 473–6

[23] Jahandideh M, Sarmin N H, Omer S M S, Ahvaz I and Benghazi L 2015 The topological indices of non-commuting graph of a finite group *Int. J. Pure Appl. Math.* **105** 27–38

[24] Jahandideh M, Darafsheh M R and Shirali N 2015 Computation of topological indices of non-commuting graphs *Ital. J. Pure Appl. Math.* 299

[25] Abdussakir 2019 Some topological indices of subgroup graph of symmetric group *Math. Stat.* **7** 98–105

[26] Abdussakir, Susanti E, Hidayati N and Ulya N M 2019 Eccentric distance sum and adjacent eccentric distance sum index of complement of subgroup graphs of dihedral group *J. Phys. Conf. Ser.* **1375** 1–6

[27] Abdussakir, Puspitasari L A, Irawan W H and Alisah E 2019 Eccentric connectivity index of identity graph of cyclic group and finite commutative ring with unity *J. Phys. Conf. Ser.* **1375** 1–6

[28] Darafsheh M R 2010 Computation of topological indices of some graphs *Acta Appl. Math.* **110** 1225–35

[29] Qu H and Cao S 2015 On the adjacent eccentric distance sum index of graphs *PLoS One* 1–12

[30] Ilic A, Yu G and Feng L 2011 On the eccentric distance sum of graphs ☆ *J. Math. Anal. Appl.* **381** 590–600

[31] Azari M and Iranmanesh A 2013 Computing the eccentric-distance sum for graph operations *Discret. Appl. Math.* **161** 2827–40

[32] Padmapriya P and Mathad V 2017 The eccentric-distance sum of some graphs *Electron. J. Graph Theory Appl.* **5** 51–62

[33] Ghorbani M and Hosseinzadeh M A 2012 A new version of Zagreb indices *Filomat* **26** 93–100

[34] Idrees N, Saif M J, Rauf A and Mustafa S 2017 First and second Zagreb eccentricity indices of thorny graphs *Symmetry (Basel).* **9** 1–9

[35] Ramane H S, Manjalapur V V and Gutman I 2017 General sum-connectivity index, general product-connectivity index, general Zagreb index and coindices of line graph of subdivision graphs *AKCE Int. J. Graphs Comb.* **14** 92–100

[36] Furtula B and Gutman I 2015 A forgotten topological index *J. Math. Chem.* **53** 1184–90

[37] Furtula B, Gutman I, Vukicevic Z K, Lekishvili G and Popivoda G 2015 On an old/new degree–based topological index *Sci. Math.* 19–31

[38] Kulli V R 2018 Computing the F-ve-degree index and its polynomial of dominating oxide and regular triangulate oxide networks *Int. J. Fuzzy Math. Arch.* **16** 1–6

[39] Hosseinzadeh M A, Iranmanesh A and Došlić T 2013 On the Narumi-Katayama index of composite graphs *Croat. Chem. Acta* **86** 503–8

[40] Li X and Shi Y 2008 A survey on the Randic index *MATCH Commun. Math. Comput. Chem* **59** 127–56

[41] Xing R, Zhou B and Dong F 2011 On atom-bond connectivity index of connected graphs *Discret. Appl. Math.*

[42] Zhong L 2012 The harmonic index for graphs *Appl. Math. Lett.* **25** 561–6

[43] Liu J 2013 On the Harmonic Index of Triangle-Free Graphs *Appl. Math.* **04** 1204–6

[44] Zhong L and Xu K 2013 The harmonic index for bicyclic graphs *Util. Math.* **90**

[45] Li J and Shiu W C 2014 The harmonic index of a graph *Rocky Mt. J. Math.* **44** 1607–20

[46] Furtula B, Graovac A and Vukičević D 2010 Augmented Zagreb index *J. Math. Chem.* **48** 370–80

[47] Das K C, Gutman I and Furtula B 2011 On the first geometric-arithmetic index of graphs *Discret. Appl. Math.* **159** 2030–7

[48] Lučić B, Trinajstić N and Zhou B 2009 Comparison between the sum-connectivity index and product-connectivity index for benzenoid hydrocarbons *Chem. Phys. Lett.* **475** 146–8

[49] Zhou B and Trinajstić N 2010 On general sum-connectivity index *J. Math. Chem.* **47** 210–8

[50] Eliasi M, Raeisi G and Taeri B 2012 Wiener index of some graph operations *Discret. Appl. Math.* **160** 1333–44

[51] Hua H and Das K C 2016 On the Wiener polarity index of graphs *Appl. Math. Comput.* **280**

[52] Xu K, Liu M, Ch Das K, Gutman I and Furtula B 2014 *A Survey on Graphs Extremal with Respect to Distance–Based Topological Indices* vol 71

[53] Ramane H S and Manjalapur V V 2015 Reciprocal Wiener index and reciprocal complementary

Wiener index of line graphs *Indian J. Discret. Math* **1** 23–32

[54] Zhou B, Cai X and Trinajstić N 2008 On Harary index *J. Math. Chem.* **44** 611–8

[55] Xu K and Das K C 2011 On Harary index of graphs *Discret. Appl. Math.* **159** 1631–40

[56] Das K C, Xu K and Gutman I 2013 On Zagreb and Harary indices *Match* **70** 301–14

[57] De N, Abu Nayeem S M and Pal A 2015 Total Eccentricity Index of the Generalized Hierarchical Product of Graphs *Int. J. Appl. Comput. Math.* **1** 503–11

[58] De N, Pal A and Nayeem S M A 2015 Total eccentricity index of some composite graphs *Malaya J. Mat* **3** 523–9

[59] HUA H 2009 Wiener and Schultz molecular topological indices of graphs with specified cut edges *MATCH Commun. Math. Comput. Chem.* **61** 643

[60] Andova V, Dimitrov D, Fink J and Škrekovski R 2012 Bounds on Gutman index *Match-Communications Math. Comput. Chem.* **67** 515

[61] Philipose R S and Sarasija P B 2018 Gutman Index and Harary Index of Unitary Cayley Graphs *Int. J. Eng. Technol.* **7** 1243–4

[62] Alizadeh Y, Iranmanesh A and Došlić T 2013 Additively weighted Harary index of some composite graphs *Discrete Math.* **313** 26–34

[63] Deng H, Krishnakumari B, Venkatakrishnan Y B and Balachandran S 2015 Multiplicatively weighted Harary index of graphs *J. Comb. Optim.*

[64] Xu K, Wang J, Das K C and Klavžar S 2015 Weighted Harary indices of apex trees and k-apex trees *Discret. Appl. Math.* **189**

[65] An M and Xiong L 2015 Multiplicatively weighted harary index of some composite graphs *Filomat*

[66] Akbari S and Mohammadian A 2004 On the zero-divisor graph of a commutative ring *J. Algebr.* **274** 847–55

[67] Barati Z, Khashyarmanesh K, Mohammadi F and Nafar K 2012 On the associated graphs to a commutative ring *J. Algebr. its Appl.* **11** 1–15

[68] Duric A, Jevdenic S, Oblak P and Stopar N 2018 The total zero-divisor graph of commutative rings *Int. J. Algebr. Its Appl.*

[69] Akbari S, Alizadeh F and Khojasteh S 2014 Some results on cozero-divisor graph of a commutative ring *J. Algebr. Its Appl.* **13** 1350113

[70] Anderson D F and Badawi A 2013 The generalized total graph of a commutative ring *J. Algebr. Its Appl.* **12** 1250212

[71] Bishop A, Cuchta T, Lokken K and Pechenik O 2008 The nilradical and non-nilradical graphs of commutative rings *Int. J. Algebr.* **2** 981–94

[72] Dutta S and Lanong C 2017 On annihilator graphs of a finite commutative ring *Trancaction Comb.* **6** 1–11

[73] Gaur A and Sharma A 2013 Maximal Graph of a Commutative Ring *Int. J. Algebr.* **7** 581–8

[74] Wang H-J 2009 Co-maximal graph of non-commutative rings *Linear Algebra Appl.* **430** 633–41

[75] Al-Kaseasbeh S 2016 On Containment and Radical Graphs of a Commutative Ring *J. Math. Stat.* **12** 308–11