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Three Magical Words of Mathematics

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Abstract

In teaching the preparation of a lecture plan is very important in the sense that in which order the chapter (or subject) must be planned and taught, so that the students can easily grasp the subject matter as well as the curriculum can be completed within fixed time period to get the optimum outcome. Generally a mathematics teacher follows three magical words in teaching from primary school to colleges and universities for primary to graduate and postgraduate level students as well as for research scholars for doctoral and post doctoral works. These three magical words are: assumptions, properties and applications. Assumptions give the base for the development of the chapter (subject), the properties provide the base for their applications and the applications show the importance of the chapter (subject). These three words are the foundations of all subjects of arts, commerce, science, management, engineering, etc. and rule the complete education, the human civilizations, and the universe.

Key Words: Axioms, Theorem, Property, Applications, Definitions, Approaches to Teaching.

AMS Subject Classification 2020: 97C70, 97D30, 97D40

Introduction

The importance of mathematics can be understood by the statement given by **Carl Friedrich Gauss** that *it is the queen of science*. This subject is very essential to the growth of many other disciplines. The science of mathematics depends on the mental ability. It is the means to develop the thinking power and reasoning intelligence, which sharps the mind and makes it creative. The development of human beings and their culture depend on the development of mathematics. This is why, it is known as the base of human civilization and it plays an important role in human life. It is also the language of all material science and the centre of all engineering branches which revolve around it. **Hodanova & Nocar** (2016) states that Mathematics is used in many fields like the study of nature, various forms of technology, architecture, the construction industry, banking, scientific research, cartography, etc. Mathematics is the past, present and future of all sciences.

In the era of modern technology the educators and policy makers of education system realized the importance of mathematics during the cold war between USA and USSR. **Narlikar** (2013) has stated that in 1957 when the Soviet Union launched the first satellite Sputnik, the United States realized that to match it, *the teaching of mathematics had to receive boost*. After that many major steps have been taken to improve the quality education of mathematics not only in USA but in the world too.

Edwin et al (2020) state that by enhancing the capabilities of human mind, mathematics has facilitated the development of science, technology, engineering, business, and government. He says that mathematics nurtures the power of reasoning, creativity, abstract or spatial thinking, problem-solving ability, critical thinking, and effective communication skills. He points the importance of mathematics by saying that *citizens who cannot reason mathematically, are cut off from whole realms of human endeavour*. He states that mathematical learning is associated with the development of mathematical understanding.

Taylor (2013) expresses that mathematical understanding entails understanding of mathematical concepts. **Simon** (2018) views that raising or increasing mathematical understanding is one of the most difficult challenges in teaching. **Simon** (2017) defined mathematical concept to be the knowledge of the mathematical necessity of a particular mathematical relationship. Mathematical concepts are the ideas that serve as a guide to a better understanding of mathematics. **Edwin et al** (2020) state that helping children to understand mathematical concepts can take variety of forms. These varieties of forms are called strategies, which is a plan of action designed to achieve the long term aim.

Rosikhoh et al (2019) state the importance of mathematics in STEAM learning. They say that in STEAM based learning, the teacher will have a challenge about how to encourage students to be able to use their understanding and logic actively, think critically and creatively and use problem solving skills. They expect from the teachers that they must be able to utilize the available time for STEAM based learning. Obviously this is not possible without mathematics, a tool to develop the thinking power.

Yakman (2008) expresses that in learning mathematics, the implementation of STEAM can be done by understanding the relationship between existing mathematical concepts with other scientific disciplines contained in STEAM. **Herro & Quigley** (2017) state that STEAM implementation in mathematics can be done well, when the teachers have understood the meaning of implementation.

Basar (2021) proposes in-service training for the teachers before the implementation of the teaching-learning process. He also supports that good relation between mathematics teacher and students positively affect the attitudes of the students toward mathematics. **Ozdemir et al** (2019) determined that students' anxiety towards mathematics was mostly related to teaching. One of the factors that cause students to worry about mathematics is the teacher. **Koca** (2011) and **Senturk** (2010) found that satisfaction with the teachers affect students' anxiety about mathematics. Being satisfied with the teachers and having a positive attitude toward the teachers, students are positively affected towards mathematics. If the students like the mathematics teacher, they may like the subject mathematics too.

Demir et al (2010) argue that the success of students in Mathematics depends on the quality of education they received during the courses. The quality of education in mathematics can only be ensured with effective curriculums. Therefore the development of mathematics also depends on the teacher, teaching and the effective curriculum. **Yadav** (2018) states the action of a person who teaches is teaching. It is a process of transferring knowledge from teachers to students using different methods i. e., it is a process of imparting knowledge or skills. **Smith** sees it as certain tasks the intention of which is to induce learning. **Castelino & Hegde** (1999-20) say that teaching is a purposeful activity carried out by the teacher for guiding, directing and helping the learners in their pursuit of realizing the set teaching-learning objectives.

There is a difference of teaching in school, college and university level students. **Raghavan** (2018) states this difference in the principles and methods of teaching, known as pedagogy. **Keiler** (2018) states that the role of a teacher has changed from teacher centered to student centered and the skills needed for qualified teachers have also changed.

Formal teaching tasks include preparing lessons, giving lessons, and assessing students. **Weimer** states that *when teachers think the best, they improve their teaching by developing their content of knowledge, they end up with sophisticated levels of knowledge, and they have simplistic instructional methods to convey that knowledge*. He stresses on *what we teach and how we teach it are inextricably linked and very much dependent on one another. The best teachers do know their material, but they also know a lot about the process*. They never underestimate the power of the process to determine the outcome. **Kelton** proposes to write up syllabi by simply handling all of the mandated material first and *prepare a general list of the material to be covered*.

As far as mathematician's approach to teaching is concerned, lecturing remains the prime delivery mode for teaching. **Yadav** (2017) states that teaching depends on the basics required for the development of the subject. He defined *mathematics as the study of assumptions, its properties and applications*. The three terms assumptions, properties and applications give the clue of teaching. He suggested that we must maintain the order of assumptions, properties and applications in teaching.

Ronning (2008) raised three questions '*What should we emphasize when we teach mathematics? What kind of understanding do we want the students to develop? What kind of mathematics, and how much, do all students need to know*?' **Kreyszig** (2005) states that it would make no sense to overload students with all kinds of little things that might be of occasional use. Instead it is important that students should be familiar with ways to think mathematically, recognize the need for applying mathematical methods, realize that mathematics is a systematic science built on relatively few basic concepts and involving powerful unifying principles, and get a firm grasp for the interrelation between theory, computing and experiment.

Yadav (2017) stated that *every chapter should be divided into three parts: assumptions, properties and applications*. He stated that during teaching, we must discuss "*what are the basic assumptions in the chapter*" keeping in view that definition is itself an assumption. What can we obtain from the assumptions and in last but not the least how and where can we apply these concepts? Following this students learn the definition, formulae and understand the basic structure of the chapter, which makes them perfect in application and for research. In this way our motives become more and more successful in increasing the interest of mathematics among students.

Preliminary Terms of Mathematics

In mathematics we always find some general terms like axioms, properties, theorem, examples, applications, etc. These are the building blocks of the subjects whose meanings are as follows:

Axioms: In mathematics dictionary **James & James** (2001) state that the axioms of a subject are the basic propositions from which all other propositions can be derived. They are the starting points and are accepted true without any proof. With the help of axioms we conclude that whether a given mathematical statement is true or false. It is also known as assumptions or hypothesis or postulates or propositions. One right angle is equal to 90^0 is an assumption. It is possible to draw a straight line from any point to any other point is an assumption.

Property: Any mathematical statement derived from axioms is known as a property. A straight angle is equal to 180^0 is a property. The sum of four angles of a square is 360^0 is a property. Property is divided in two parts: Theorem and Conjecture.

Theorem: A general conclusion proved by the help of axioms is called a theorem. In general it is proved logically using assumptions. The sum of the three angles of a triangle is 180^0 is a theorem.

Conjecture: A mathematical statement which is yet to be proved is known as conjecture. A well known conjecture is *Goldbach's Conjecture* which states that e*very even integer greater than 4 can be expressed as sum of two odd primes*.

Applications: If we apply the assumptions and their related properties to solve real world problems, we say that such type of assumptions and properties have applications. The sum of the three angles of a triangle is 180^0 is an application of the assumption that one right angle is 90^0 and the property that a straight angle is 180^0.

Analysis of Definition of Mathematics

Research in mathematics has covered a milestone but a drawback has been seen in the literature of mathematics that it could not be defined properly. According to **Wikipedia**, *mathematics has no generally accepted definition*. In **German Wikipedia** it is written that *there is no definition of mathematics, or at least no commonly accepted one*. The famous book by **Richard Courant** and **Herbert Robbins** (1996) entitled "What is Mathematics?" (and subtitled "An Elementary Approach to Ideas and Methods") does not give a satisfactory answer about definition of mathematics. They claim that *it is impossible to give a good definition of mathematics in a sentence or two*. Majority of professional mathematicians did not take any interest in a definition of mathematics.

Traditionally *it is defined as the scientific study of quantities, including their relationship, operations and measurements expressed by numbers and symbols*. In mathematics dictionary by **James & James** (2001) *it has been defined as the science of logical study of numbers, shape, arrangement, quantity, measure and many related concepts*. According to **Wikipedia**, *Mathematics is the study of quantity, structure, space*. Mathematics seeks out patterns and uses them to formulate new conjectures. **Aristotle** defined mathematics as *the science of quantity*. **Benjamin Pierce** defined it as m*athematics is the science that draws necessary conclusions*. **Haskell Curry** defined mathematics simply as *the science of formal systems*. **Albert Einstein** stated that *as far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality*, and so on. All most all great mathematicians stated something for it but no generally accepted definition could be propounded. More recently **Yadav** (2017) defined *Mathematics as the study of assumptions, its properties and applications*. He claimed that all mathematical and non-mathematical subjects are the study of assumptions, its properties and applications. To get the better understanding of three words assumptions, properties, and applications, let us have a look at the next analysis.

Analysis of Branches of Mathematics

Generally mathematics is grouped into two main branches: pure and applied. In pure mathematics we study about assumptions and properties, where as in applied mathematics we study a little about assumptions and properties but more on their applications. Mathematics is also branched into four parts: Arithmetic, Algebra, Analysis and Geometry.

According to **wikipedia** arithmetic deals with numbers, the basic operations - addition, subtraction, multiplication, division, and relations between them. They are assumptions and the properties. Algebra is an extension part of arithmetic where we use unknown quantities along with numbers. These unknown quantities are represented by letters of the English alphabets or symbols. These help to generalize the formulae and rules and also help to find the unknown quantities in the algebraic expressions and equations. So in it the assumptions and properties of arithmetic are used as applications. In higher algebra we use basic properties like closure, associative, commutative, etc. which are only assumptions and assumed properties. Analysis deals with the study of the rate of change in different quantities, which are assumptions and their properties. Geometry deals with spatial relationship using fundamental axioms. These axioms are used in conjunction with mathematical definition of points, straight lines, curves, surfaces, and solids to draw logical conclusions. In fact geometry starts from Euclid's assumptions and its contradictions.

Thus we analyze that in all pure and applied mathematics, arithmetic, algebra, analysis, and geometry, we study about assumptions, their properties and applications.

Analysis of Approaches to Teaching and Learning Mathematics

Castelino & Hegde (1999-20) state that approaches serve as a guiding force to teaching-learning process. Approaches help the teachers to select the right methods and techniques in teaching, which give the optimum learning experience to the pupils. Approaches chosen in teaching mathematics have to help the pupils to create and discover their own knowledge with the help of teaching-learning techniques.

Inductive and deductive approaches are nothing but a game between assumptions and properties. In inductive method a rule is constructed through particular to general, whereas in deductive the rule is generated through general to particular. So these methods are surrounded by assumptions, properties and applications. The same happens with analytic and synthetic approaches. In analytic method we start with the unknown, and try to find possibilities which may connect with the known to find out the desired result. In synthetic method we start with the known and connect it to the unknown to find out the desired result. Therefore these approaches are also surrounded by assumptions, properties and applications.

In constructivist approach people create meaning through a series of individual experiences. In it learning environment is provided where students can explore, test and acquire knowledge on their own. **Yadav** (2017, 2018) states that if we can teach the lesson in sequence following assumptions, properties and applications,

means we are providing them such learning environment and compelling them to generate the knowledge of their own. Such approach makes the true learner a *dummy creator* of the subject. Therefore constructivism is nothing special but the strategies made by the teacher and provided to the students to learn and think of self created lesson and knowledge.

In his realistic method **Yadav** (2018) suggested only three subtopics: assumptions, properties, and applications, which is more realistic method than other approaches under the constraint of limited time period allowed for each subject. Realistic method contains both inductive method and deductive method strategies. In idealistic method (also called research oriented method) he suggested for research oriented students of doctorate and post doctoral work for that a teacher needs deep knowledge of the subject and the chapter. In it he suggested that the teacher must prepare the lectures containing: Historic Background, Basics, Assumptions and Definitions, Properties, Applications, Interdisciplinary Connections, Limitations, Philosophy and Social Values, Critical Thinking, Current Research, and Further Scope of Research in the subject to Generate New Knowledge. In this method all other points except the three words assumptions, properties, applications are also surrounded by those three exceptions words.

Conclusion

From introduction to analysis, we conclude that mathematics and its teaching learning process are surrounded by only three words assumptions, properties and applications. These words rule the complete education, the human civilization and the real and imaginary worlds i.e., the universe. Without any hesitation, it can be generalized that these words are the foundations of all subjects of arts, science, commerce, management, engineering, etc. Therefore these three magical words are the past, present, and future of all scientific and non-scientific subjects from past to future.

Open Problems

All subjects have only three magical words and can be verified by studying different approaches to teaching and the basics propounded for the development of that particular subjects. The research on this is open to all true research scholars in all fields of arts, science, commerce, management, engineering, etc.

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